

# SPHINCS: practical stateless hash-based signatures

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Where innovation starts

# Digital Signatures are Important!



Software updates



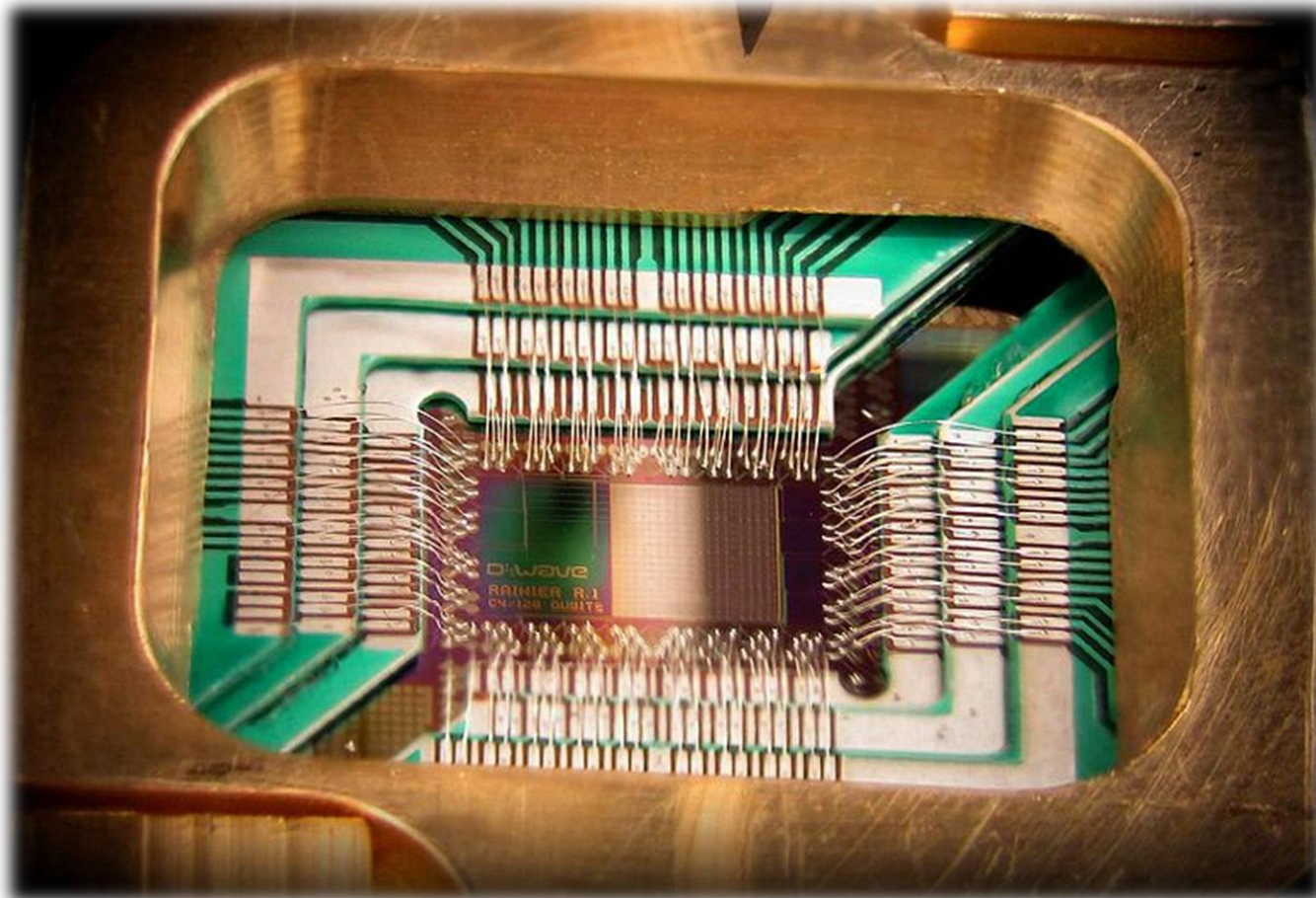
amazon

E-Commerce

ebay<sup>™</sup>

... and many others

# What if...



# Problem?

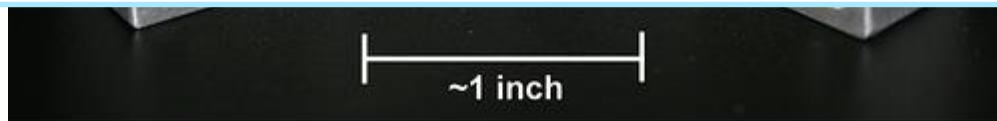


## IBM 2012: „...optimism about superconducting qubits and the possibilities for a future quantum computer are

**„Want to factor a 2000-bit number?**

- **Quantum computer runtime = 24h**
- **Number of physical qubits required = 500M**
- **Power required ~ dedicated nuclear power plant**
- **Lead time ~5 (research) + 10 (development) years**
- **Dollar required ~1 billion (CERN ~ 10 billion)“**

Matteo Mariantoni, PQ-Crypto 2014



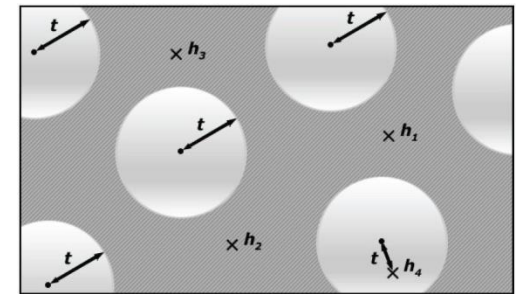
# Post-Quantum Signatures

## Lattice, MQ, Coding

 Signature and/or key sizes

 Runtimes

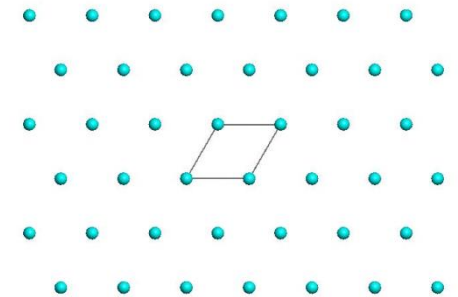
 Secure parameters



$$y_1 = x_1^2 + x_1x_2 + x_1x_4 + x_3$$

$$y_2 = x_3^2 + x_2x_3 + x_2x_4 + x_1 + 1$$

$$y_3 = \dots$$



# Hash-based Signature Schemes [Mer89]

Post quantum

Only secure hash function

Security well understood

Fast

Stateful

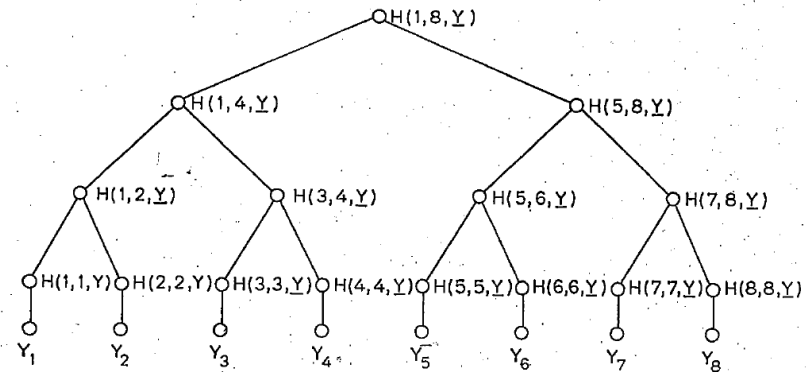


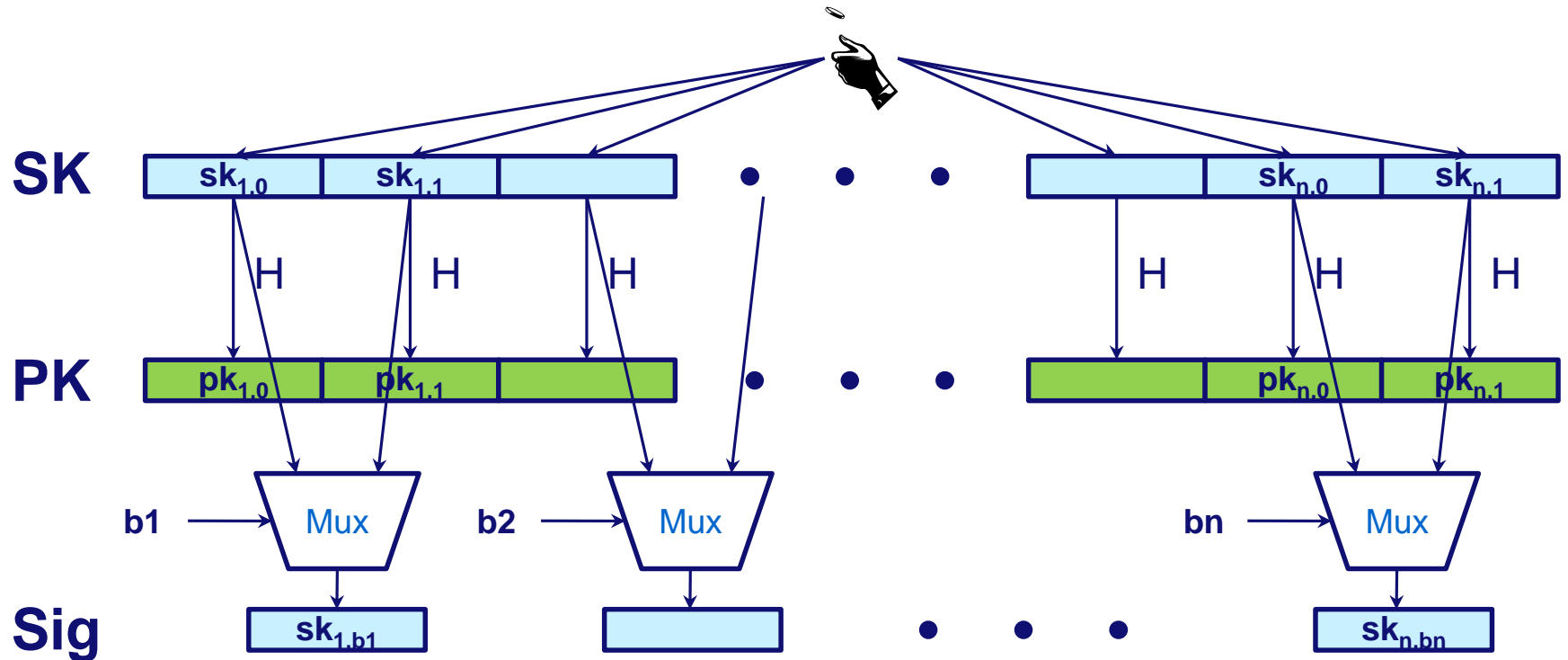
FIG 1  
AN AUTHENTICATION TREE WITH  $N = 8$ .

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# Lamport-Diffie OTS [Lam79]

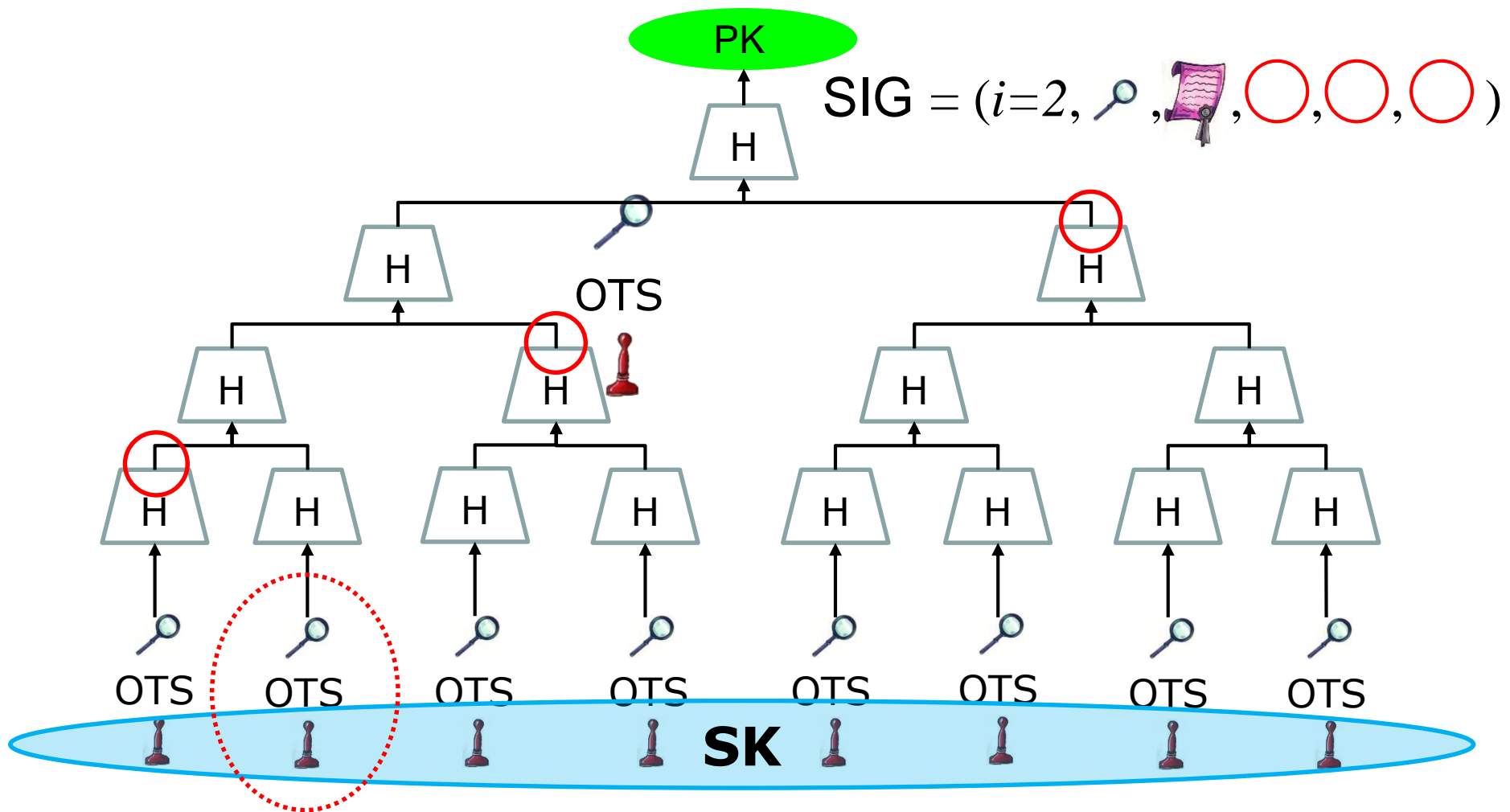
Message  $M = b_1, \dots, b_n$ , OWF  $H$

$\boxed{*}$  =  $n$  bit





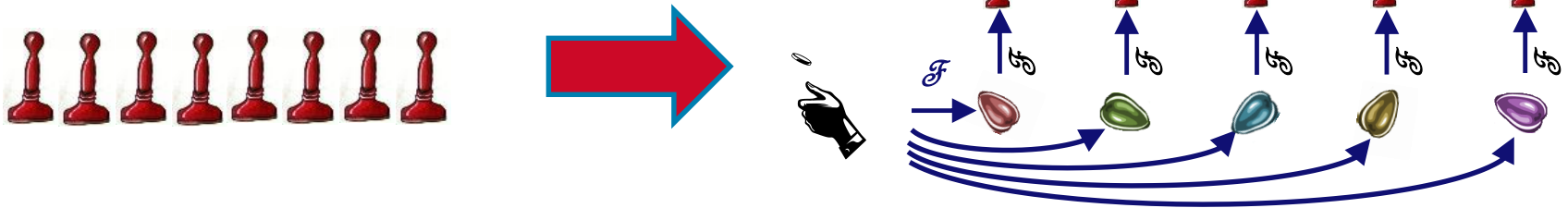
# Merkle's Hash-based Signatures



# Known Improvements [BGD+06]

- Pseudorandom Key Generation:

$$\text{SK} \quad 2^h n \rightarrow n$$

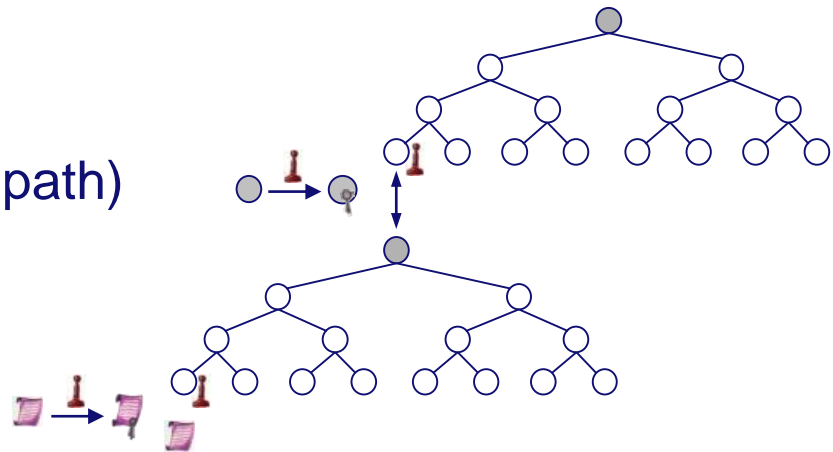


- Hypertree:

Key generation

(== Building Trees on one path)

$$\mathcal{O}(2^h) \rightarrow \mathcal{O}(d2^{h/d})$$

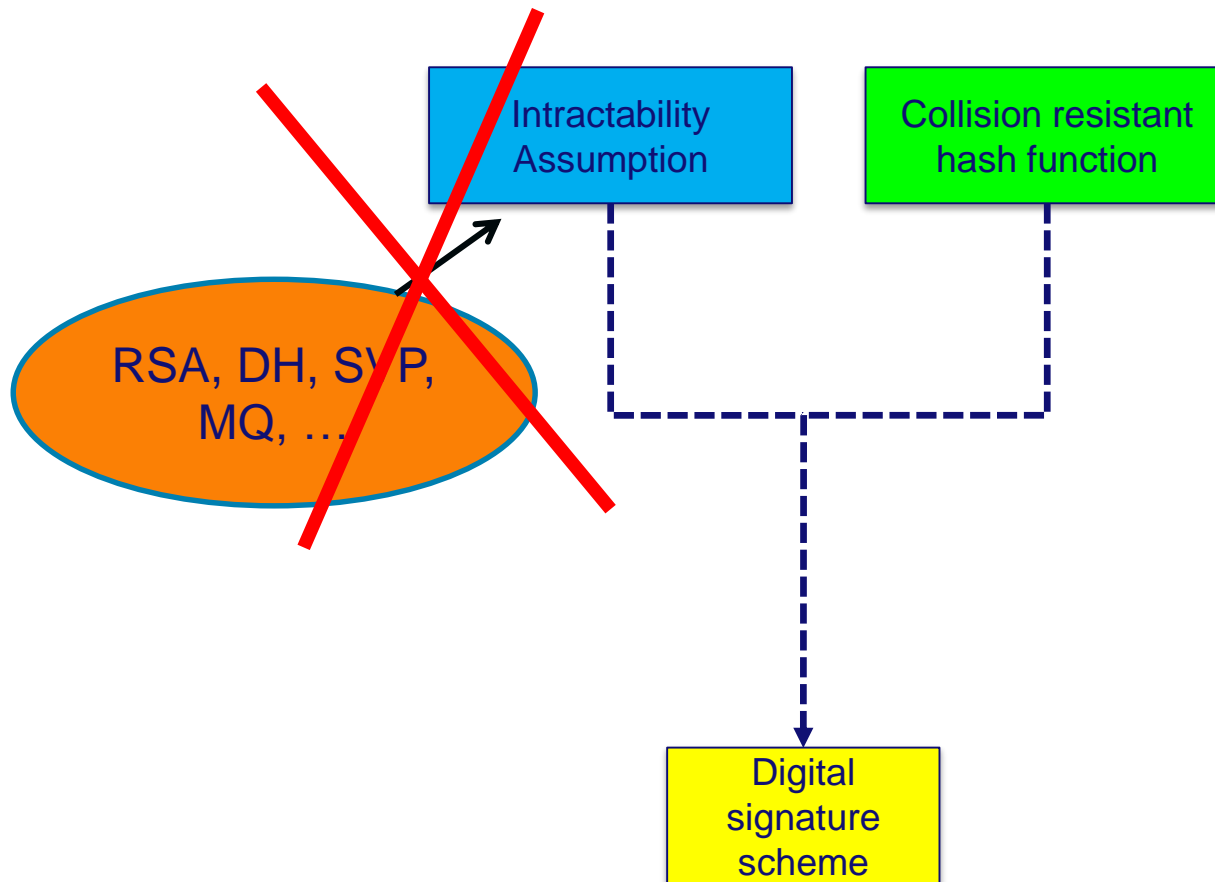


# Known Improvements, cont'd

- **Winternitz OTS [Mer'89, ..., BDE+11,Hül13]**
  - **Smaller Sigs**
  - **Minimum Security Assumptions**
  
- **Minimum Security Assumptions / Collision-Resilient Scheme [BDH11]**
  - **XMSS requires only PRF & SPR**

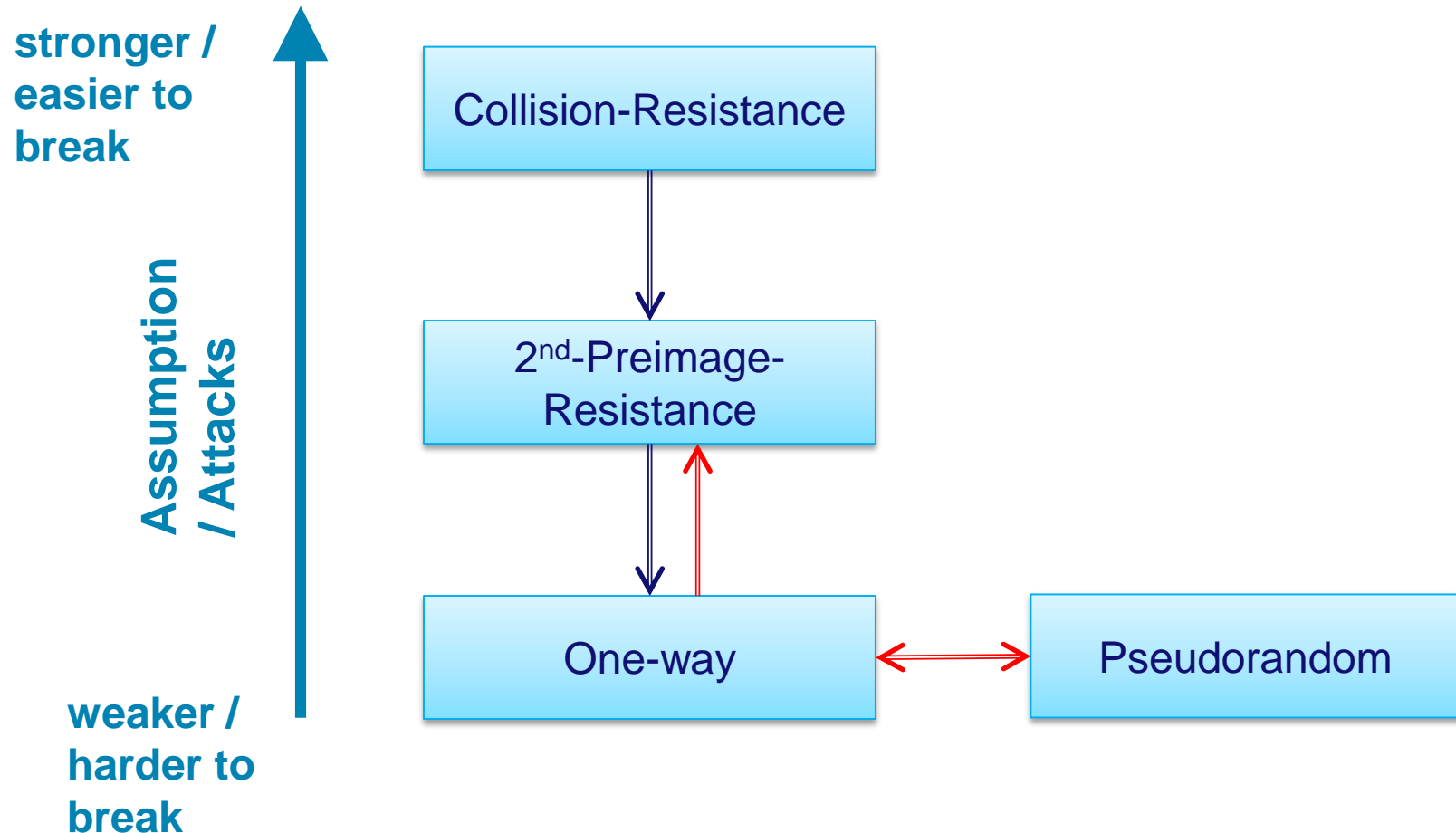
# Advantages of Hash-based Signatures

# RSA – DSA – EC-DSA...



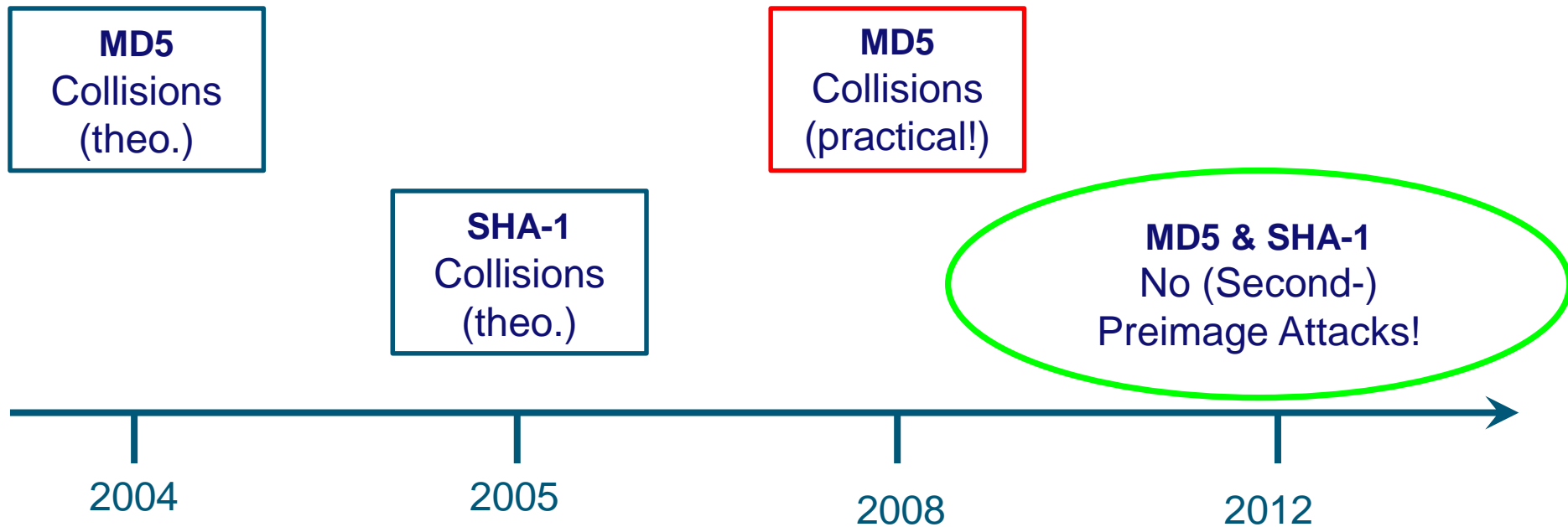
# Early Warning System

## Hash-function properties



# Early Warning System

## Attacks on Hash Functions



## Hash-Combiner

- **Collision-Resistance / 2<sup>nd</sup>-Preimage-Resistance:**

$$h_k(x) = g_k(x) \parallel f_k(x)$$

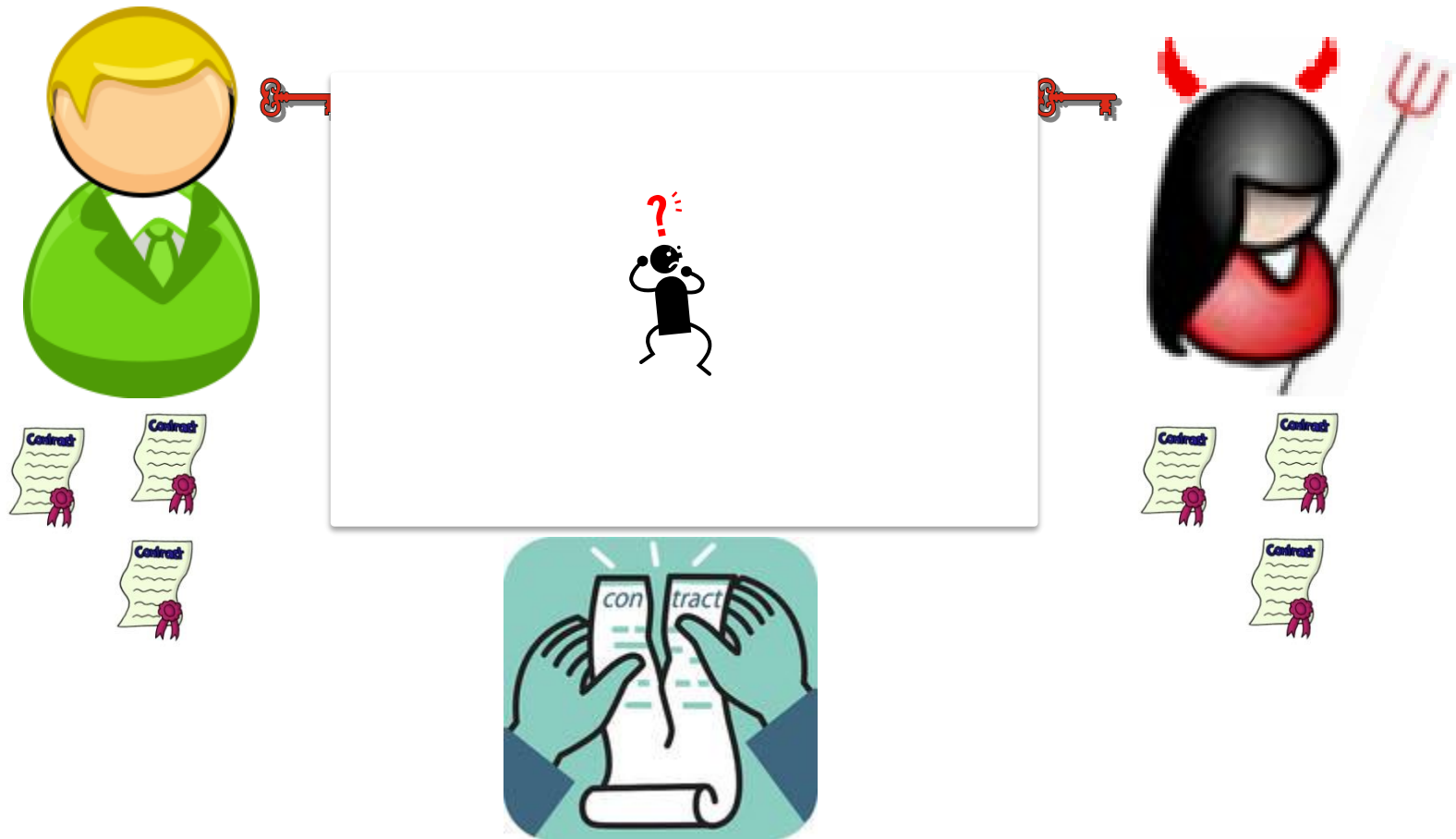
- **PRF:**

$$h_k(x) = g_k(x) \oplus f_k(x)$$

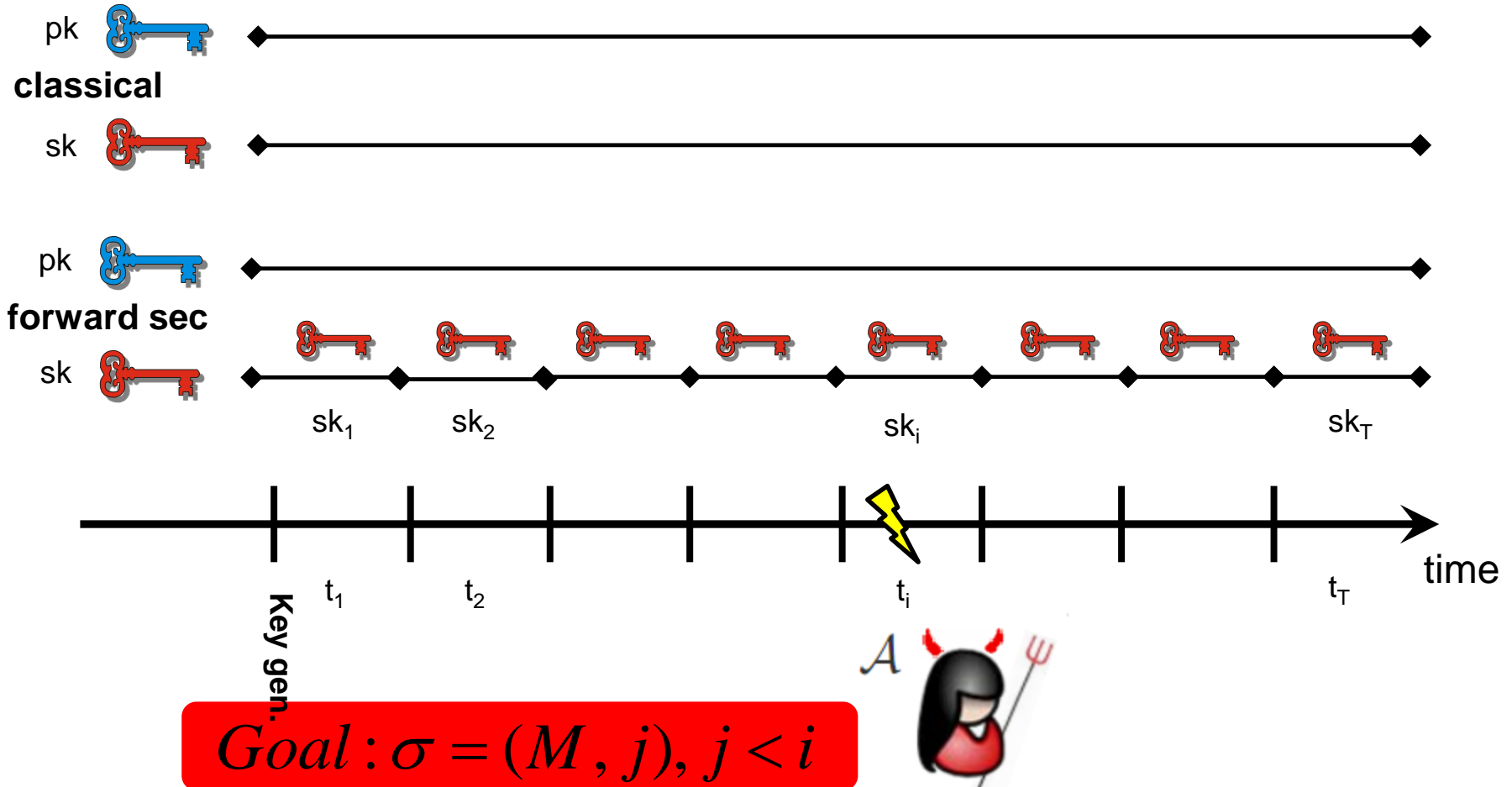
- 
- **No sudden break**
  - **Replaces double signature**
  - **Signature size only grows by  $h \cdot n$**
  - **Runtime ~ doubled**



# Forward Security



# Forward Security - cont'd



# Long-Standing Problem: Statefulness

- **No problem in many cases.**
  - Qualified signatures,
  - Keys on smartcard, ...
- **Necessary for forward-security!**



**But:**

- **Key back-ups undermine security**
- **Parallel use of key problematic**
  - Multi-threading scenarios,
  - Load balancing...
- **Do not fit standard API**



# SPHINCS: Stateless Practical Hash-based Incredibly Nice Cryptographic Signatures



# Goals

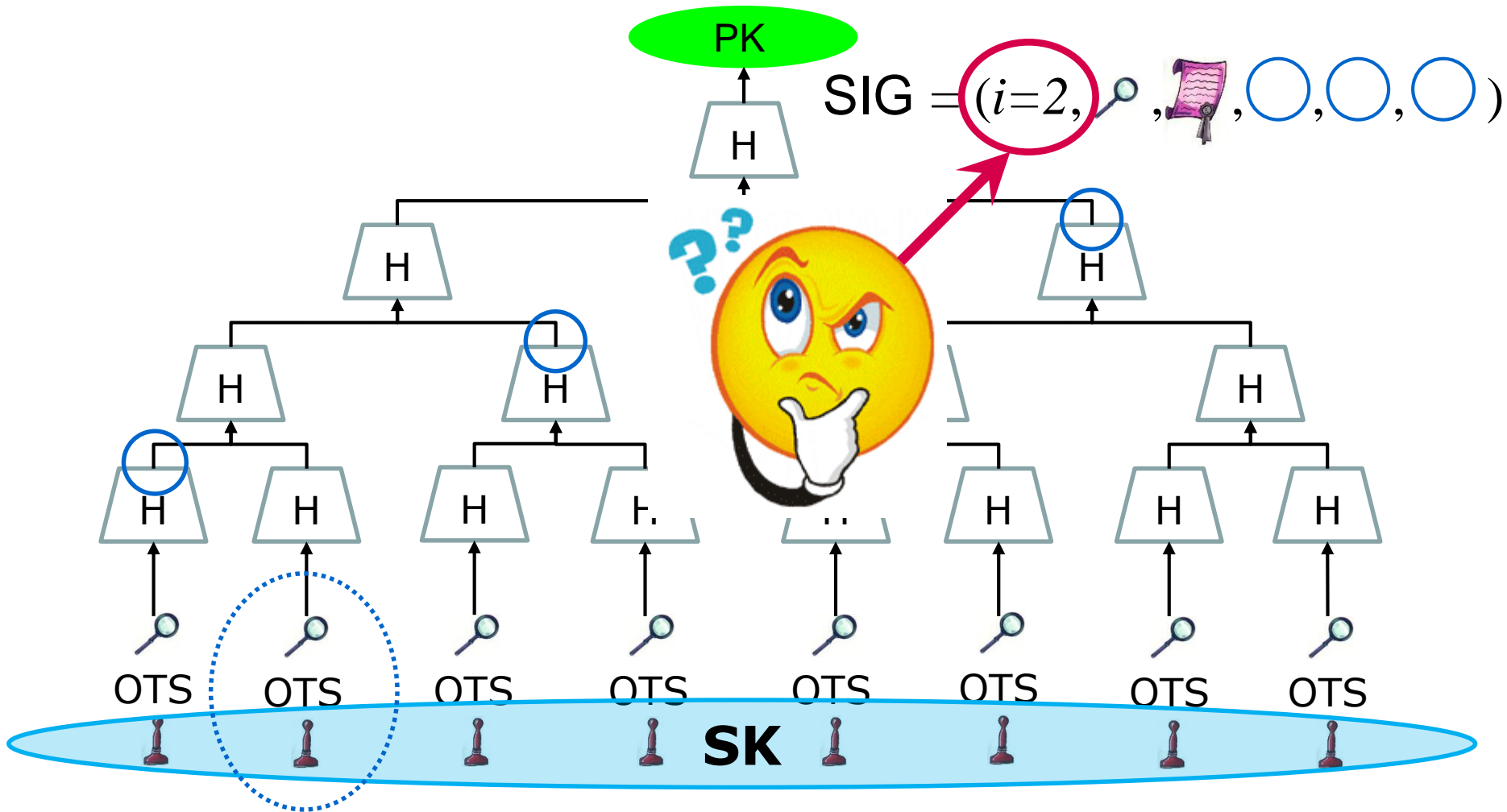
- **Stateless**
- **128bit Quantum Security**
- **Practical Speed**
- **Practical Signature Size**

# How to Eliminate the State



# Straight Forward

- Run MSS without State



# Approach 1: Message Hash

$i = \text{Hash}(\text{Message});$

- 128bit Quantum Sec.

→  $n = 256$  bit Hash [Ber09]

→ #Indices =  $2^{256}$

→  $h = n = 256$

- $h$  depends on  $n!$

- Best we can do:

$t_{\text{Sign}} \approx n^3 / \log n$   $t_{\text{Hash}} = 2M$   $t_{\text{Hash}} \approx 15 \text{ min}^*$

$|\text{Sig}| \approx n^3 / \log n > 256 \text{ kb}$

\* (OpenSSL SHA2)



# Approach 2: Random Index

$$I \leftarrow_{\$} U_{\#Indices}$$

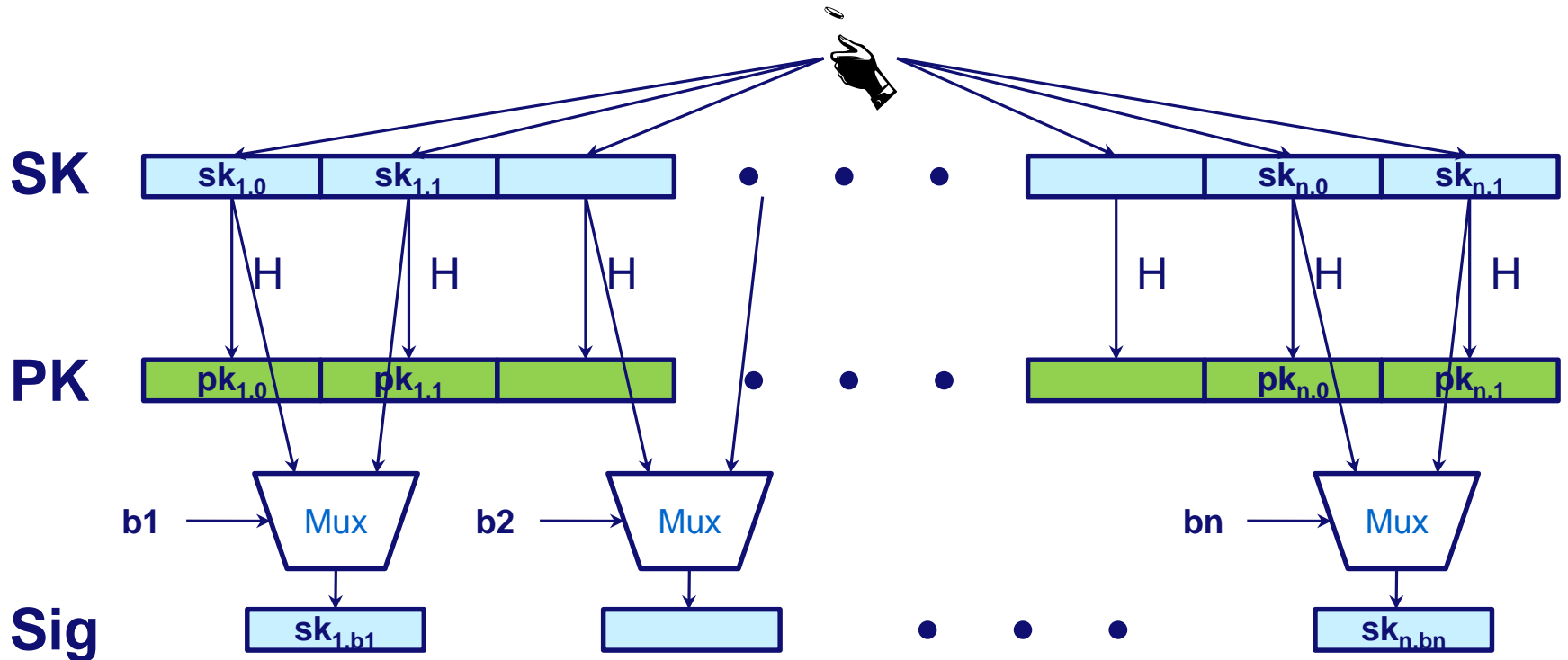
- **128bit Quantum Sec.**
  - **Sampled by Signer**
    - **#Indices determined by collision prob.**
    - **#Indices =  $2^{256}$**
    - **$h = 256$**
- **Impossible to make this efficient, again... BUT**
  - **$h$  independent of  $n$**
  - **Statistical collision probability NOT collision resistance**

# Few-Time Signature Schemes

# Recap LD-OTS

Message  $M = b_1, \dots, b_n$ , OWF  $H$

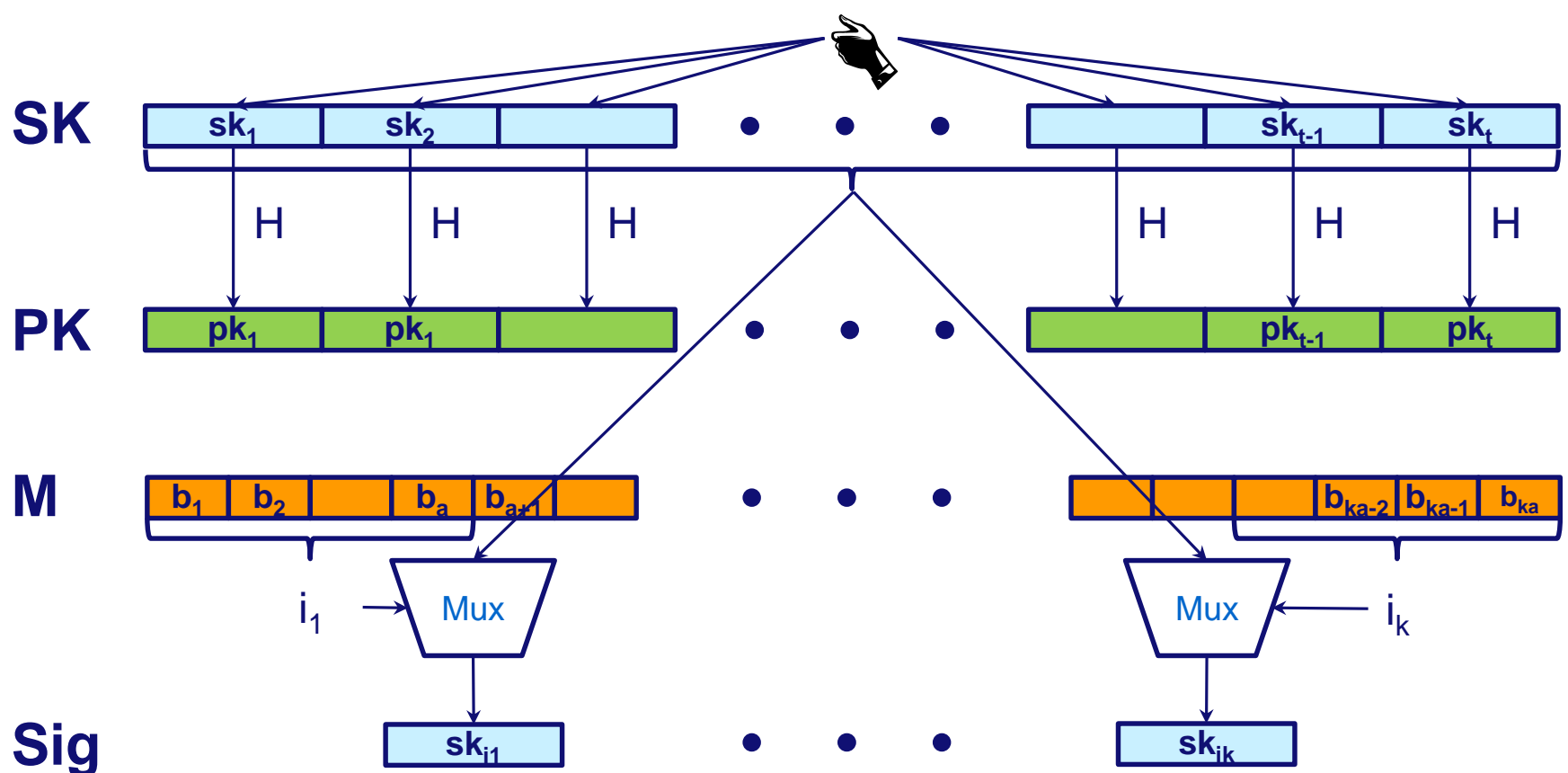
$\boxed{*}$  = n bit



# HORS [RR02]

Message  $M = b_1, \dots, b_m$ , OWF  $H$        $\boxed{*}$  =  $n$  bit

Parameters  $t=2^a, k$ , with  $m = ka$  (typical  $a=16, k=32$ )



# HORS Security

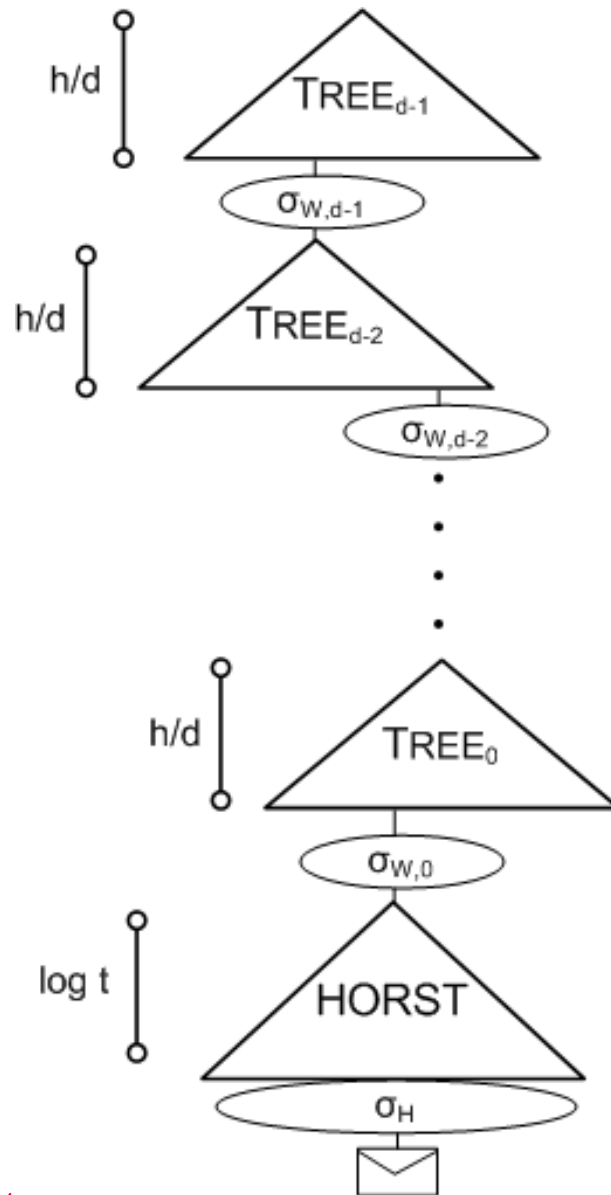
- Message  $M = \text{Hash}(\text{msg})$
- $M$  mapped to  $k$  element index set  $M^i \in \{1, \dots, t\}^k$
- Each signature publishes  $k$  out of  $t$  secrets
- Either break one-wayness or...
- **r-Subset-Resilience:** After seeing index sets  $M_j^i$  for  $r$  messages  $\text{msg}_j$ ,  $1 \leq j \leq r$ , hard to find  $\text{msg}_{r+1} \neq \text{msg}_j$  such that  $M_{r+1}^i \in \bigcup_{1 \leq j \leq r} M_j^i$ .
- **Best generic attack:**  $\text{Succ}_{r\text{-SSR}}(A, q) = q^{rk / t}^k$   
→ Security shrinks with each signature!



- Using HORS with MSS requires adding PK (tn) to MSS signature.
- **HORST: Build a Merkle Tree on top of HORS-PK**
  - PK = Root
  - Publish Authentication Paths for HORS signature values
  - PK can be computed from Sig
  - With optimizations:  $tn \rightarrow (k(\log t - x + 1) + 2^x)n$ 
    - E.g. SPHINCS-256: 2 MB  $\rightarrow$  16 KB

# SPHINCS

# SPHINCS Signature





# SPHINCS Key Ideas

- Use HORST key pairs to sign messages
- Authenticate HORST key pairs using hypertree (of MSS trees)
- Use random index

- Select Parameters such that

$$\sum_{r \in [0, \infty)} (\text{Pr}[r\text{-times index collision}] * \text{Succ}_{r\text{-SSR}}(A)) = \text{negl}(n)$$

# SPHINCS-256

**Table 1:** SPHINCS-256 parameters and functions for the 128-bit post-quantum security level and resulting signature and key sizes.

Parameter	Value	Meaning
$n$	256	bitlength of hashes in HORST and WOTS
$m$	512	bitlength of the message hash
$h$	60	height of the hyper tree
$d$	12	layers of the hyper tree
$w$	16	Winternitz parameter used for WOTS signatures
$t$	$2^{16}$	number of secret-key elements of HORST
$k$	32	number of revealed secret-key elements per HORST signature
Functions		
Hash $\mathcal{H}$ :	$\mathcal{H}(R, M) = \text{BLAKE-512}(R\ M)$	
PRF $\mathcal{F}_a$ :	$\mathcal{F}_a(A, K) = \text{BLAKE-256}(K\ A)$	
PRF $\mathcal{F}$ :	$\mathcal{F}(M, K) = \text{BLAKE-512}(K\ M)$	
PRG $G_\lambda$ :	$G_\lambda(\text{SEED}) = \text{ChaCha12}_{\text{SEED}}(0)_{0, \dots, \lambda-1}$	
Hash $F$ :	$F(M_1) = \text{CHOP}(\pi_{\text{ChaCha}}(M_1\ C), 256)$	
Hash $H$ :	$H(M_1\ M_2) = \text{CHOP}(\pi_{\text{ChaCha}}(\pi_{\text{ChaCha}}(M_1\ C) \oplus (M_2\ 0^{256})), 256)$	
Sizes		
<b>Signature size:</b>	41000 bytes	
<b>Public-key size:</b>	1056 bytes	
<b>Private-key size:</b>	1088 bytes	

# SPHINCS-256 Speed

- **Key generation:** 3,051,562 cycles
- **Verification:** 1,369,060 cycles
- **Signature:** 47,466,005 cycles
  
- **Still hundreds of messages per second on a modern 4-core 3.5GHz Intel CPU (13ms / Sig)**
  
- **Remember: Straight Forward**  
 $t_{\text{Sign}} \approx 15 \text{ min}^*$   
 $|\text{Sig}| > 256 \text{ kb}$

- + **Standard model security reduction without collision resistance**
- + **Complexity of generic quantum attacks**
- + **Efficient fixed-input length hashing**
- + **Optimized implementation**

# Conclusion

- **If you can live with a state: Go for XMSS.**
- **Otherwise:**
  - **Go for Sphincs-256!**
  - **First stateless signature scheme with post-quantum secure parameters**
  - **Practical Speed and Sizes**

# Thank you!

# Questions?

