Hash-based Signatures and SPHINCS

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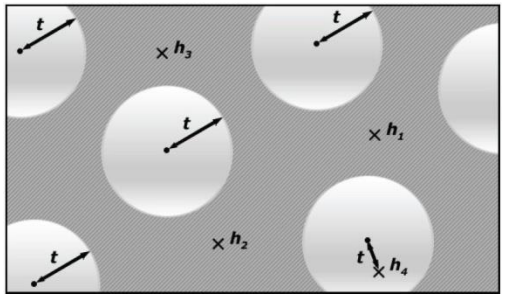
Where innovation starts

Post-Quantum Signatures

Lattice, MQ, Coding

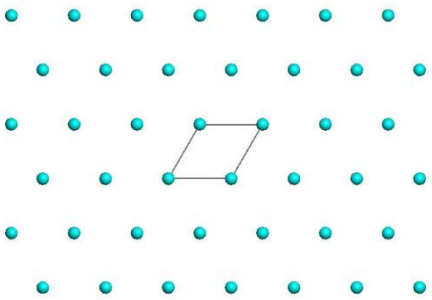
- **Signature and/or key sizes**
- **Runtimes**
- **Secure parameters**



$$y_1 = x_1^2 + x_1x_2 + x_1x_4 + x_3$$

$$y_2 = x_3^2 + x_2x_3 + x_2x_4 + x_1 + 1$$

$$y_3 = \ldots$$



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Hash-based Signature Schemes [Mer89]

- Post quantum
- Only secure hash function
- Security well understood
- Fast
- Stateful

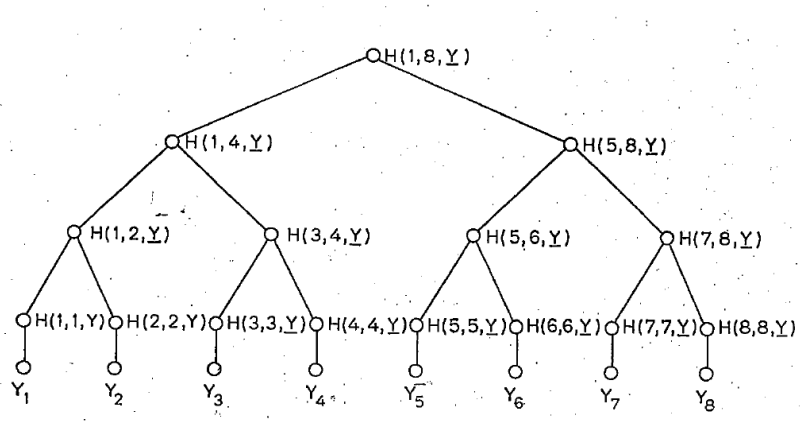

FIG 1
AN AUTHENTICATION TREE WITH N = 8.

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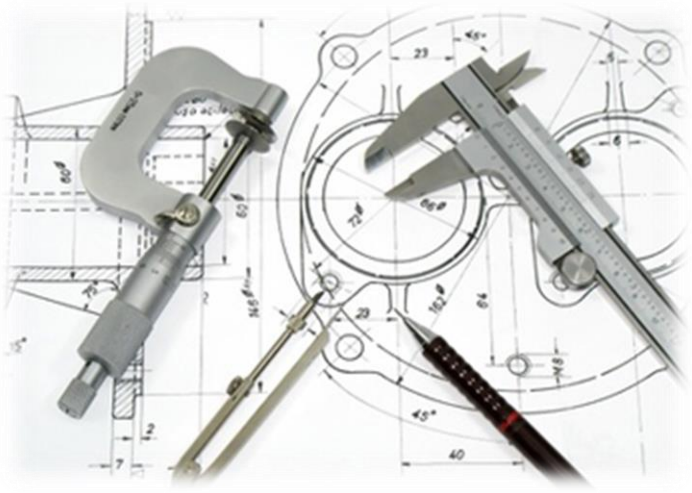
Basic Construction



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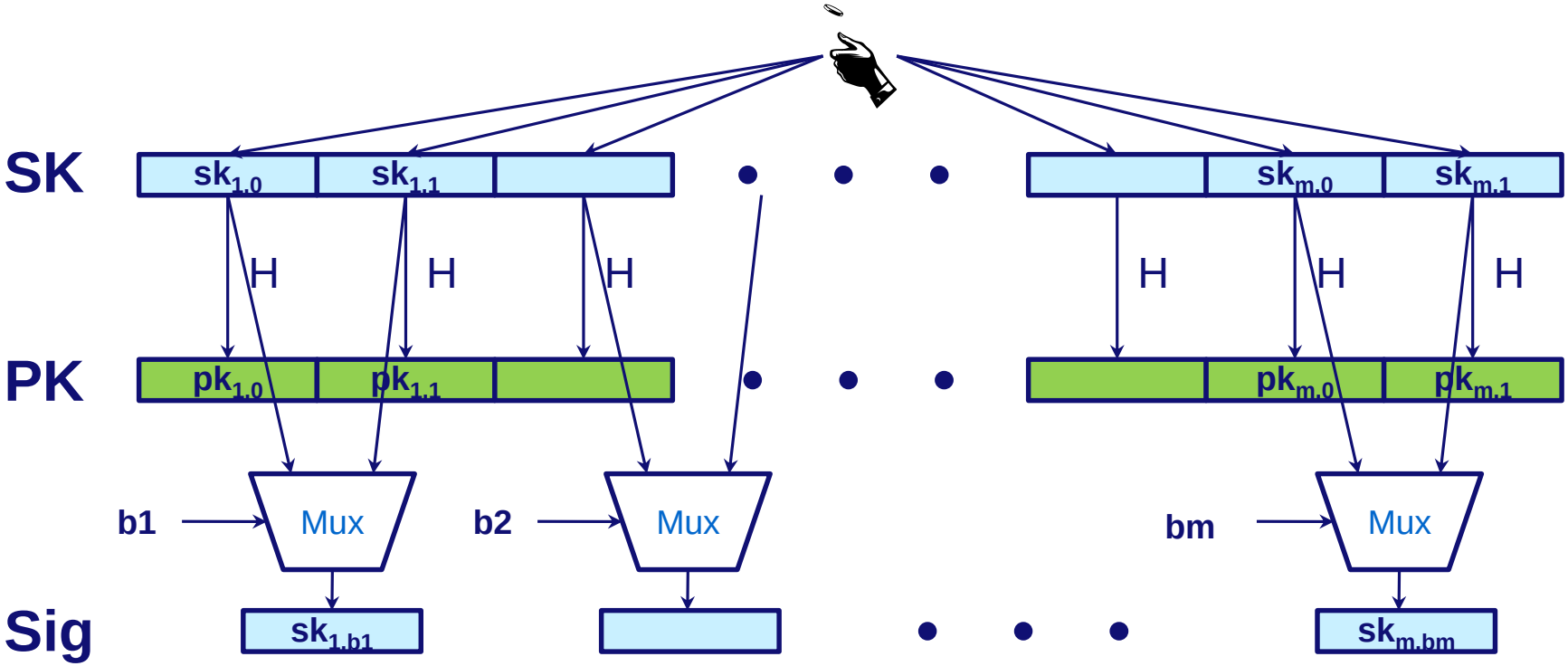
Lamport-Diffie OTS [Lam79]

Message M = b1,…,bm, OWF H * **= n bit**



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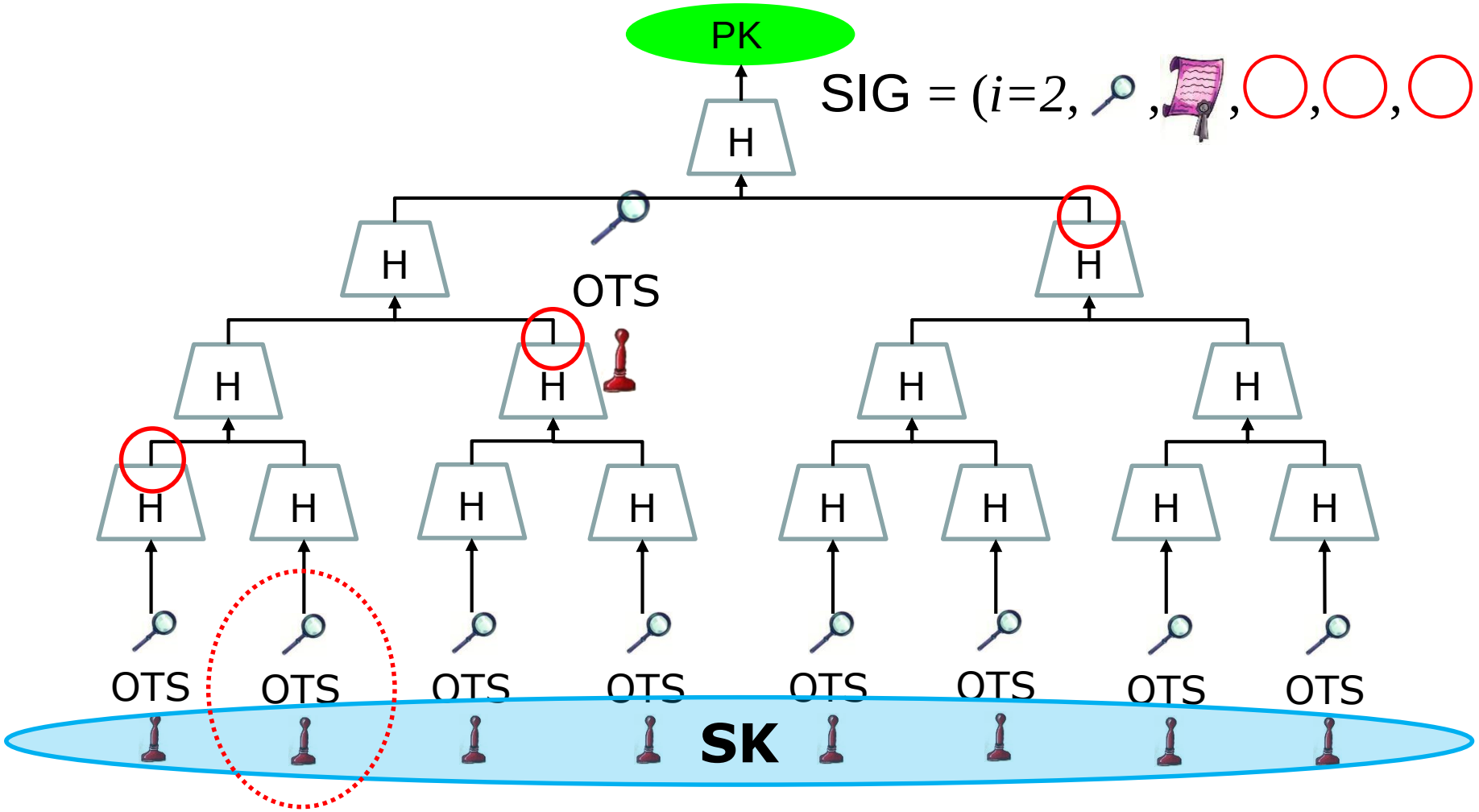
Merkle's Hash-based Signatures



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XMSS:

A practical signature scheme with minimal security assumptions

Johannes Buchmann, Carlos Coronado, Erik Dahmen, Andreas Hülsing

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XMSS Security

Security parameter n

Requires family of functions $\mathcal{F} : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$

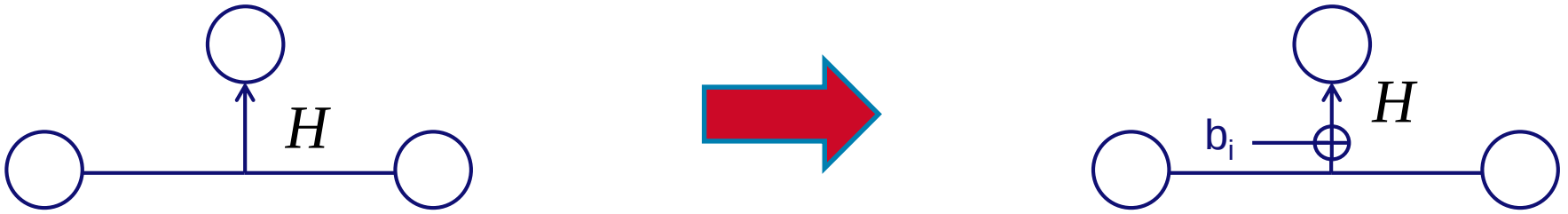
Requires family of functions $\mathcal{G} : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$

Requires family of functions $\mathcal{H} : \{0,1\}^{2n} \times \{0,1\}^n \to \{0,1\}^n$

Theorem:

XMSS is existentially unforgeable under adaptive chosen message attacks if $\mathcal{F}$ is a 2^{nd}-preimage-resistant family of undetectable one-way functions, $\mathcal{G}$ is a pseudorandom function family, and $\mathcal{H}$ is a 2^{nd}-preimage-resistant function family.

XMSS Tree

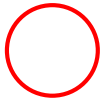


- Hashing one-time PK‘s using tree
- Requirements: CRHF -> SPRHF
- PK includes ~h additional values

XMSS uses Winternitz OTS

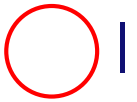
Security level b

SIG = (i, )

|  | = |  | = m * |  | = m*b

1.  = f ()

2. Trade-off between runtime and signature size

|  | ~ m/log w * |  |

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Winternitz OTS (WOTS)

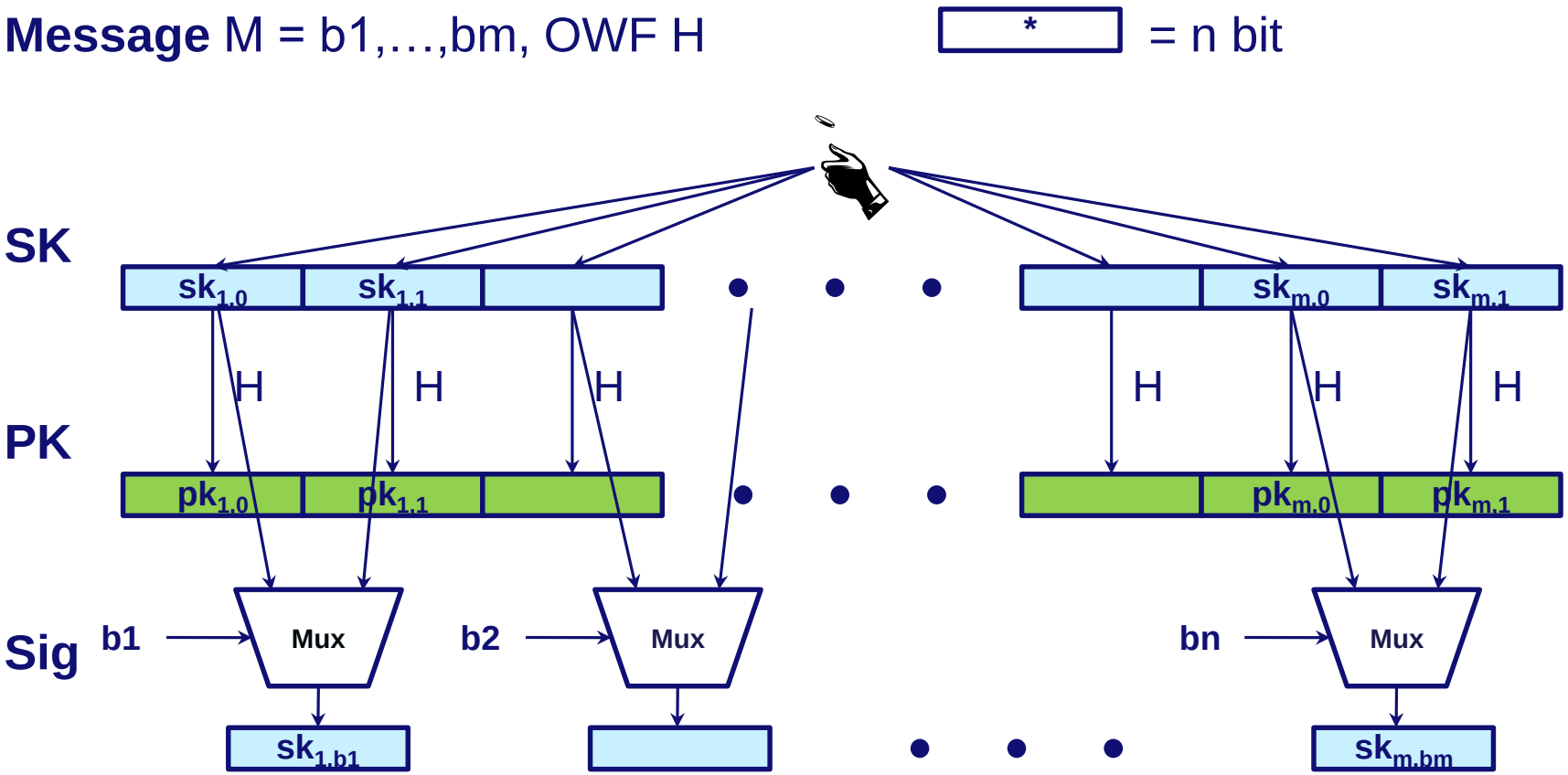
First idea: Winternitz (Mer89)
Full scheme: Even et al. (EGM96)
Security Proofs: Hevia & Micciancio (HM02)
Dods et al. (DSS05)
Requires collision-resistant undetectable one-way function family.

WOTS$^{\$}$: Buchmann et al. (BDEH^{+}11)
Requires pseudorandom function family.

WOTS^{+}: Hülsing (Hül13)
Requires second preimage resistant undetectable one-way function family.

Recap LD-OTS [Lam79]

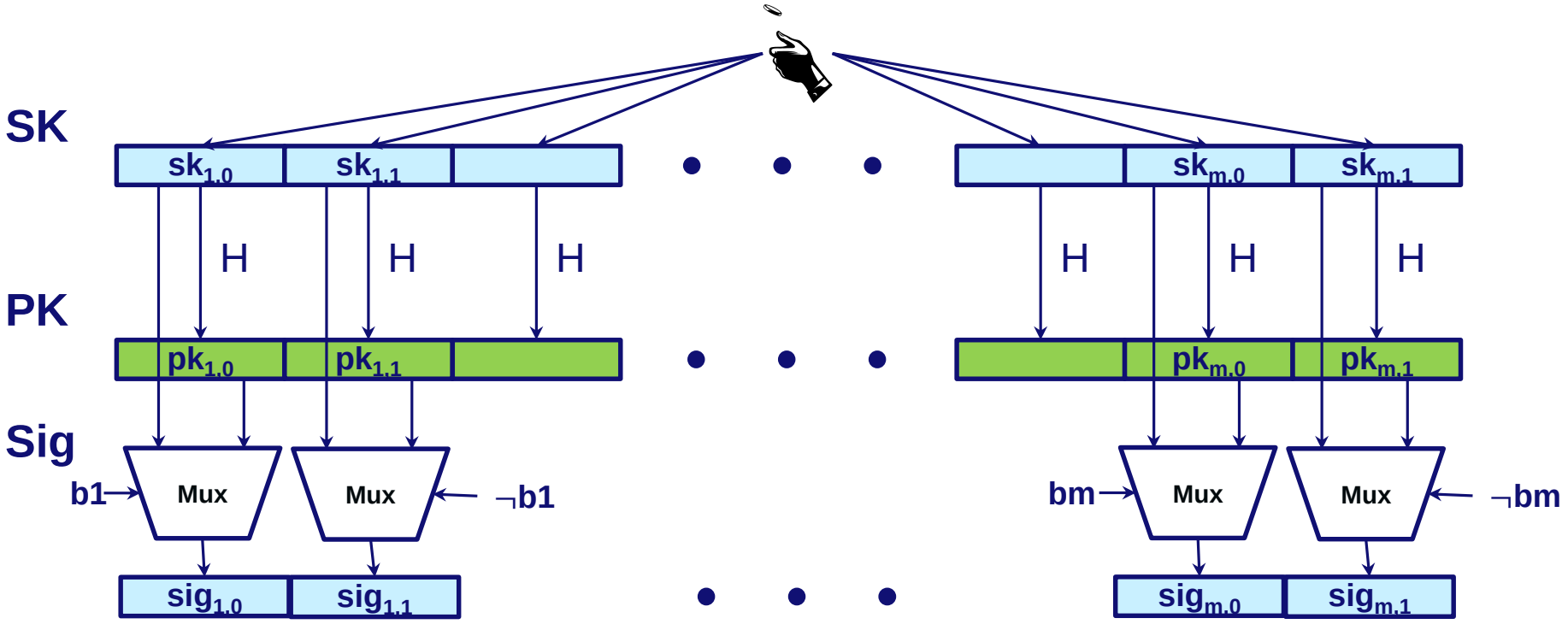
Message M = b1,…,bm, OWF H



Trivial Optimization

Message M = b1,…,bm, OWF H

* = n bit



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Non-trivial Optimization

Message $M = b_1,\ldots,b_m$, OWF H

SK: $sk_1,\ldots,sk_m,sk_{m+1},\ldots,sk_{2m}$

PK: $H(sk_1),\ldots,H(sk_m),H(sk_{m+1}),\ldots,H(sk_{2m})$

Encode M: $M' = b_1,\ldots,b_m,\neg b_1,\ldots,\neg b_m$

Sig: $sig_i =$ $\begin{cases} sk_i & \text{, if } b_i = 1 \\ H(sk_i) & \text{, otherwise} \end{cases}$

Checksum with bad performance!

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Non-trivial Optimization, cont‘d

Message M = $b_1,\ldots,b_m$, OWF H

SK: $sk_1,\ldots,sk_m,sk_{m+1},\ldots,sk_{m+\log m}$

PK: $H(sk_1),\ldots,H(sk_m),H(sk_{m+1}),\ldots,H(sk_{m+\log m})$

Encode M: M‘ = $b_1,\ldots,b_m,\neg \sum_1^m b_i$

Sig: sig_i **=**

$$\begin{cases} sk_i & \text{, if } b_i = 1 \\ H(sk_i) & \text{, otherwise} \end{cases}$$

IF one b_i is flipped from 1 to 0, another b_j will flip from 0 to 1

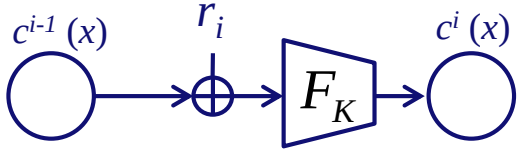
WOTS
Function Chain

Function family: $\mathcal{F}_n = \{F_K : \{0,1\}^n \rightarrow \{0,1\}^n \mid K \in \{0,1\}^{n'}\}$

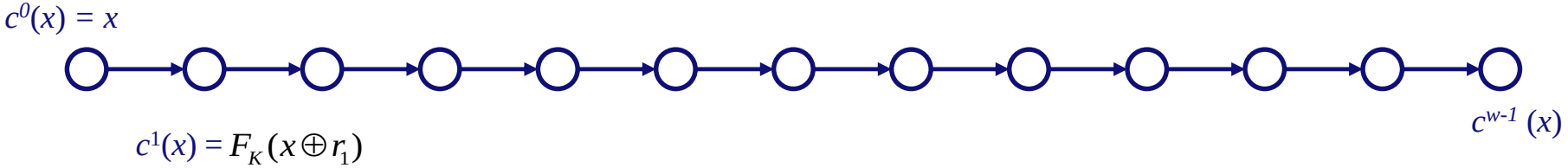
Formerly: $c^i(x) = F_K(c^{i-1}(x)) = \underbrace{F_K \circ F_K \circ \ldots \circ F_K}_{i-times}(x), K \in \{0,1\}^{n'}$

WOTS$^+$

For w $\geq$ 2 select $\mathcal{R} = (r_1, \ldots, r_{w-1}) \in \{0,1\}^{n \times w-1}, K \in \{0,1\}^{n'}$



$$c^i(x) = F_K(c^{i-1}(x) \oplus r_i)$$



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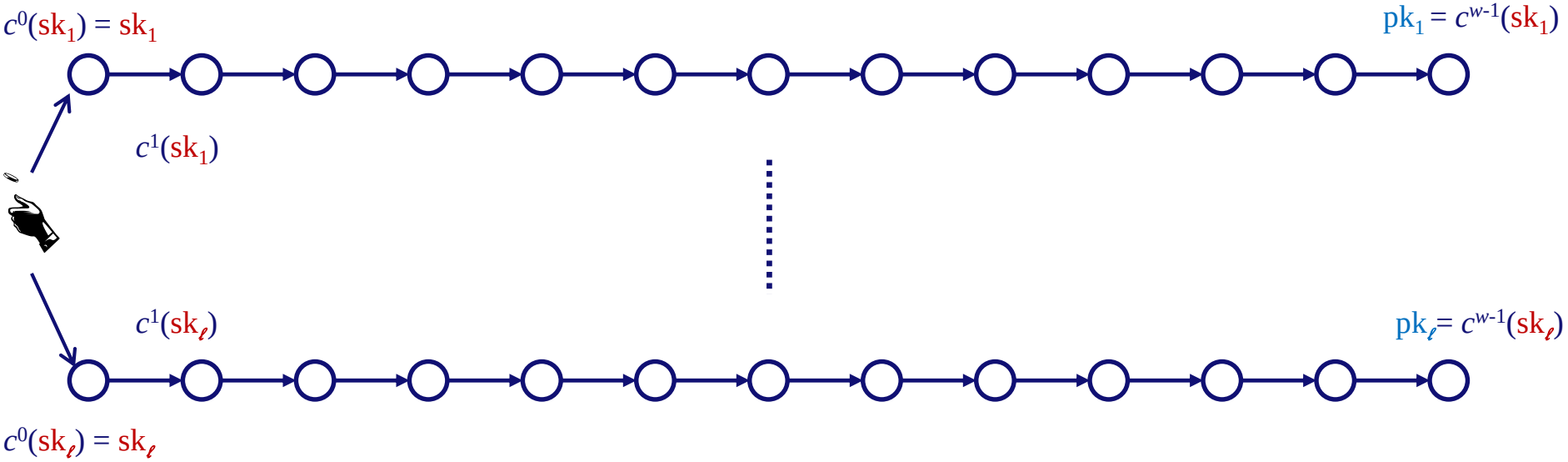
WOTS+

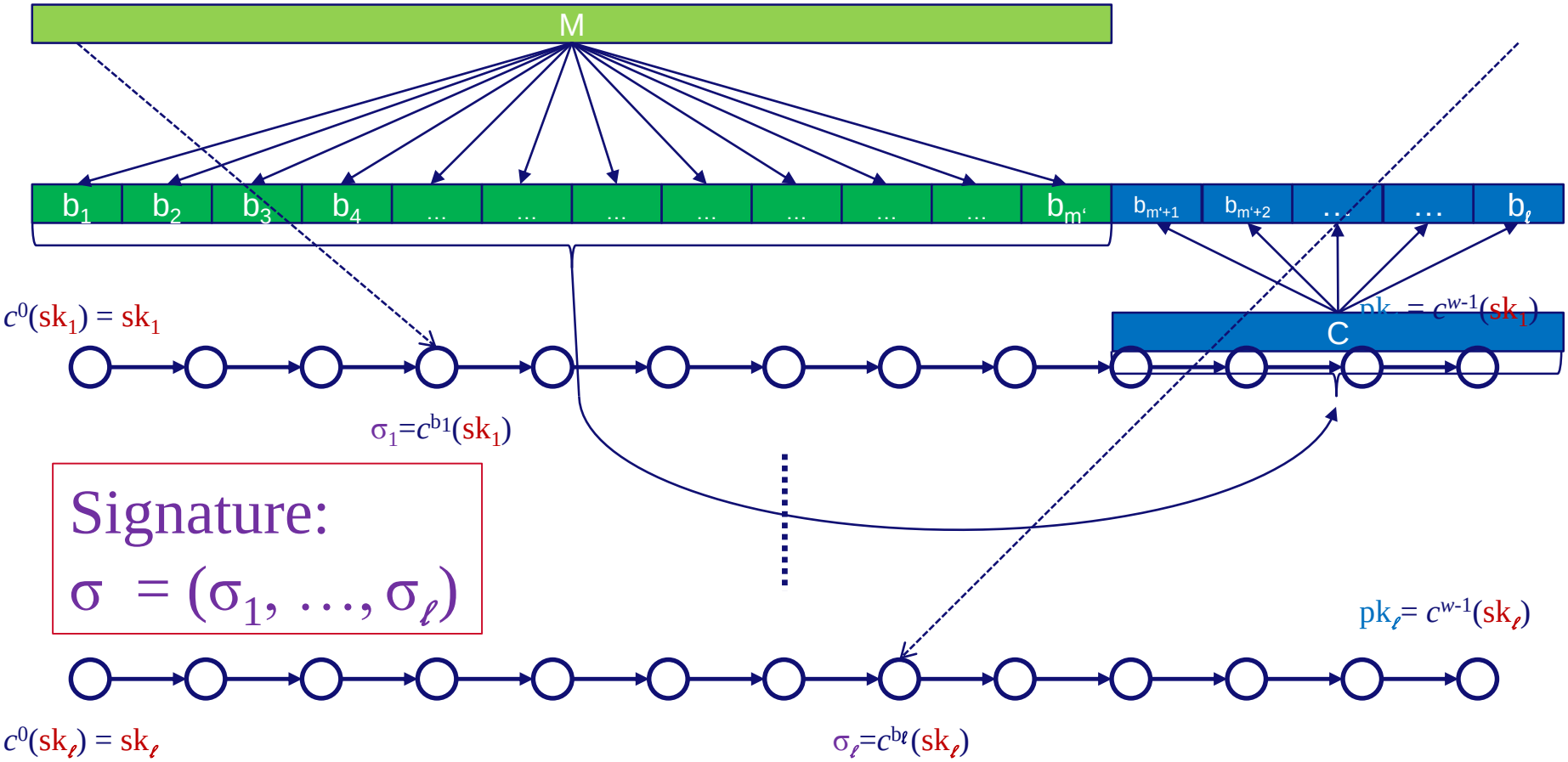
Winternitz parameter w, security parameter n, message length m, function family

$$\mathcal{F}_n = \{F_K : \{0,1\}^n \rightarrow \{0,1\}^n \mid K \in \{0,1\}^{n'}\}$$

Key Generation: Compute ℓ, sample K, sample $\mathcal{R}$



WOTS+ Signature generation



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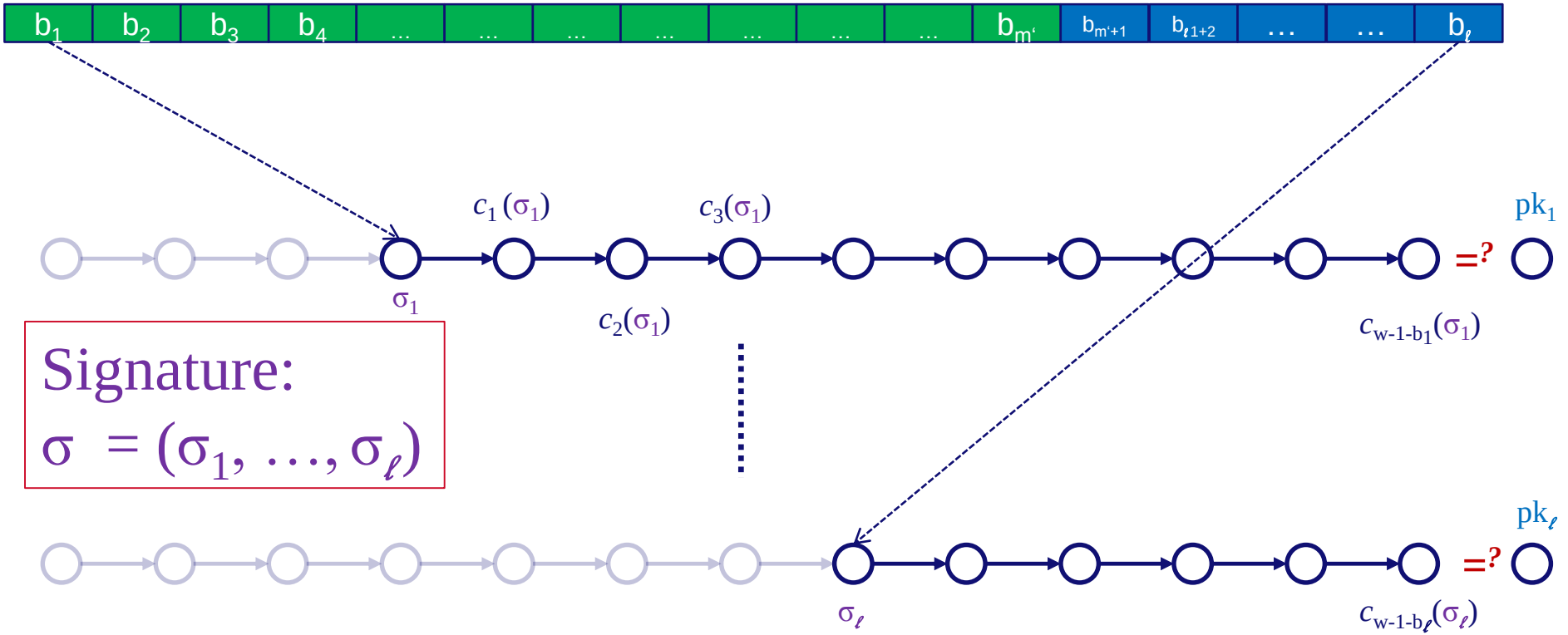
WOTS+ Signature Verification

Verifier knows: M, w



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WOTS Function Chains

For $x \in \{0,1\}^n$ define $c_0(x) = x$ and

- **WOTS:** $c_i(x) = F_K(c_{i-1}(x))$
- **WOTS$^{\$}$:** $c_i(x) = F_{c_{i-1}(x)}(r)$
- **WOTS^{+}:** $c_i(x) = F_K(c_{i-1}(x) \oplus r_i)$

WOTS Security

Theorem (informally):

W-OTS is strongly unforgeable under chosen message attacks if $\mathcal{F}$ is a collision resistant family of undetectable one-way functions.

W-OTS$^{\$}$ is existentially unforgeable under chosen message attacks if $\mathcal{F}$ is a pseudorandom function family.

W-OTS^{+} is strongly unforgeable under chosen message attacks if $\mathcal{F}$ is a 2nd-preimage resistant family of undetectable one-way functions.

WOTS Sizes and Runtimes

	Lamport-Diffie	WOTS	WOTS$^{\$}$	WOTS^{+}
Public Key Size	2bm	ℓ 2b ~ 2bm/log w	ℓ b (+b) ~ bm/log w	ℓ b (+(w-1)b) ~ bm/log w
Secret Key Size	2bm	ℓ 2b ~ 2bm/log w	ℓ b ~ bm/log w	ℓ b ~ bm/log w
Signature Size	bm	ℓ 2b ~ 2bm/log w	ℓ b ~ bm/log w	ℓ b ~ bm/log w
Key Generation Time	~ 2m	ℓ w ~ wm/log		ℓ w ~ wm/log w

WOTS$^{\$}$:
Securitly loss linear in w
-> Only small w

Security level b, Winternitz parameter w, Message Length m,
$\ell = \ell(w,m) \sim m / \log w$

Secret Key Generation

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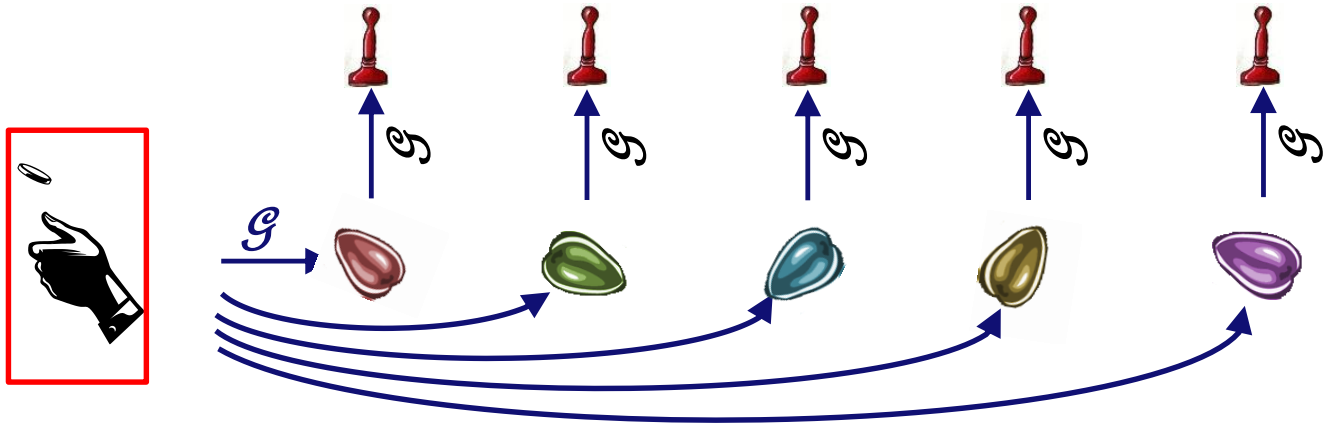
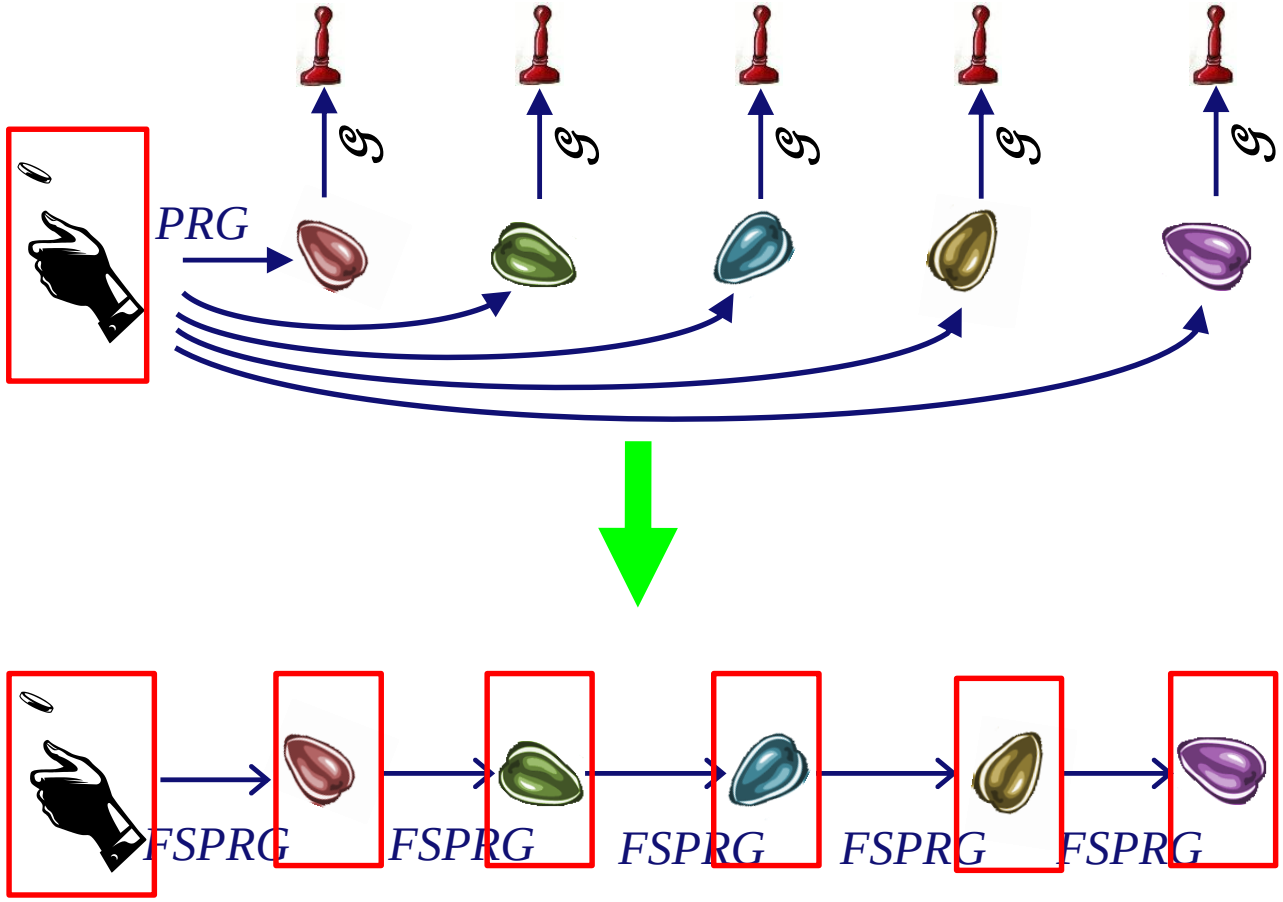
XMSS – Secret key

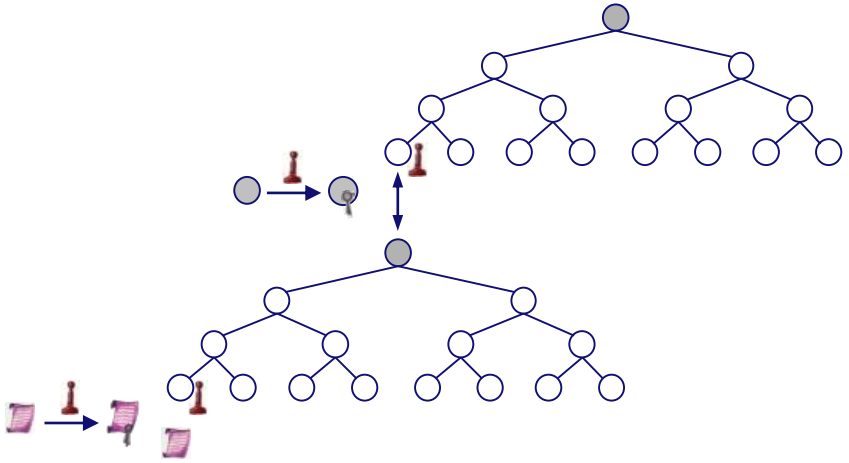


Secret Key Size: $2^{h}\ell\, b \rightarrow b$

XMSS forward secure



Tree Chaining



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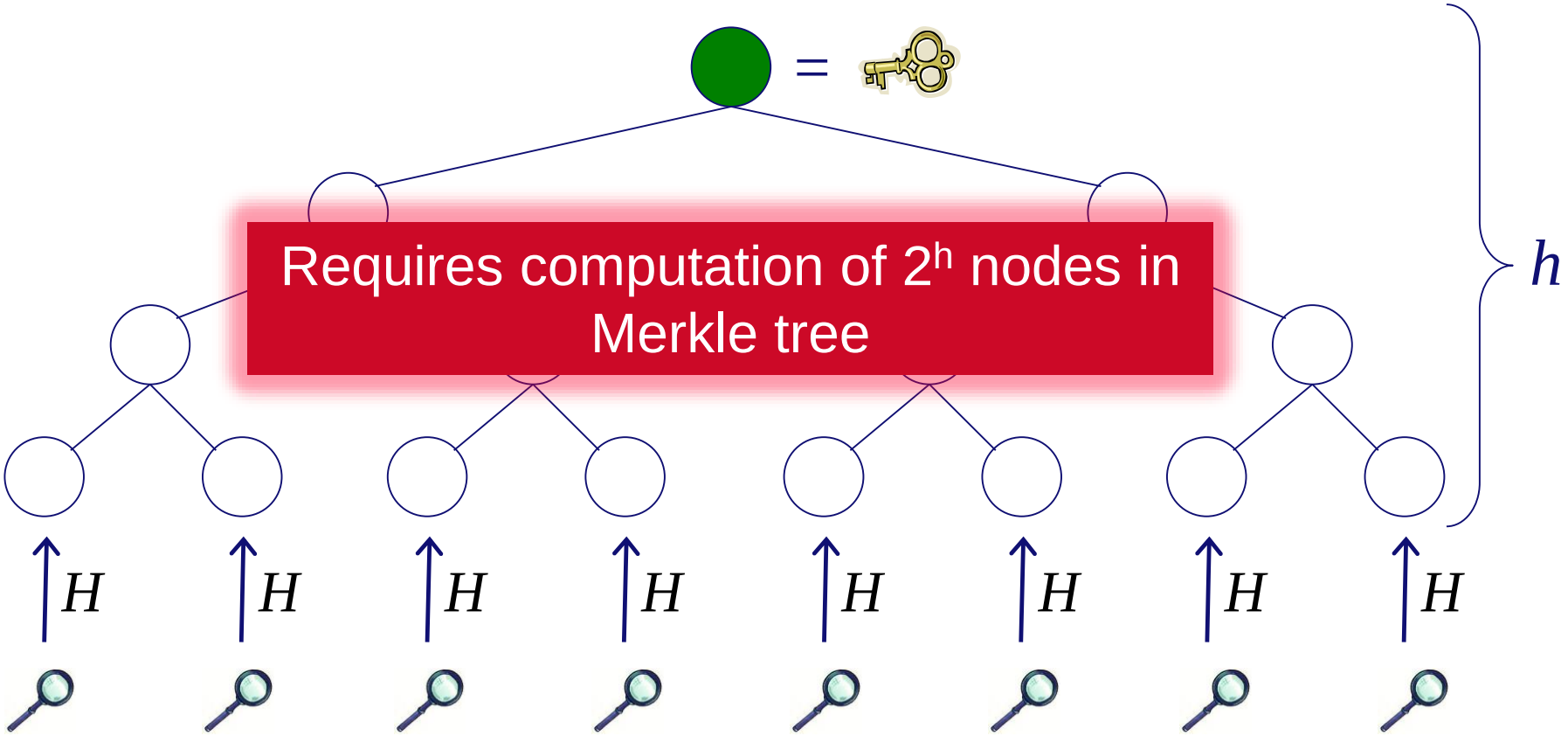
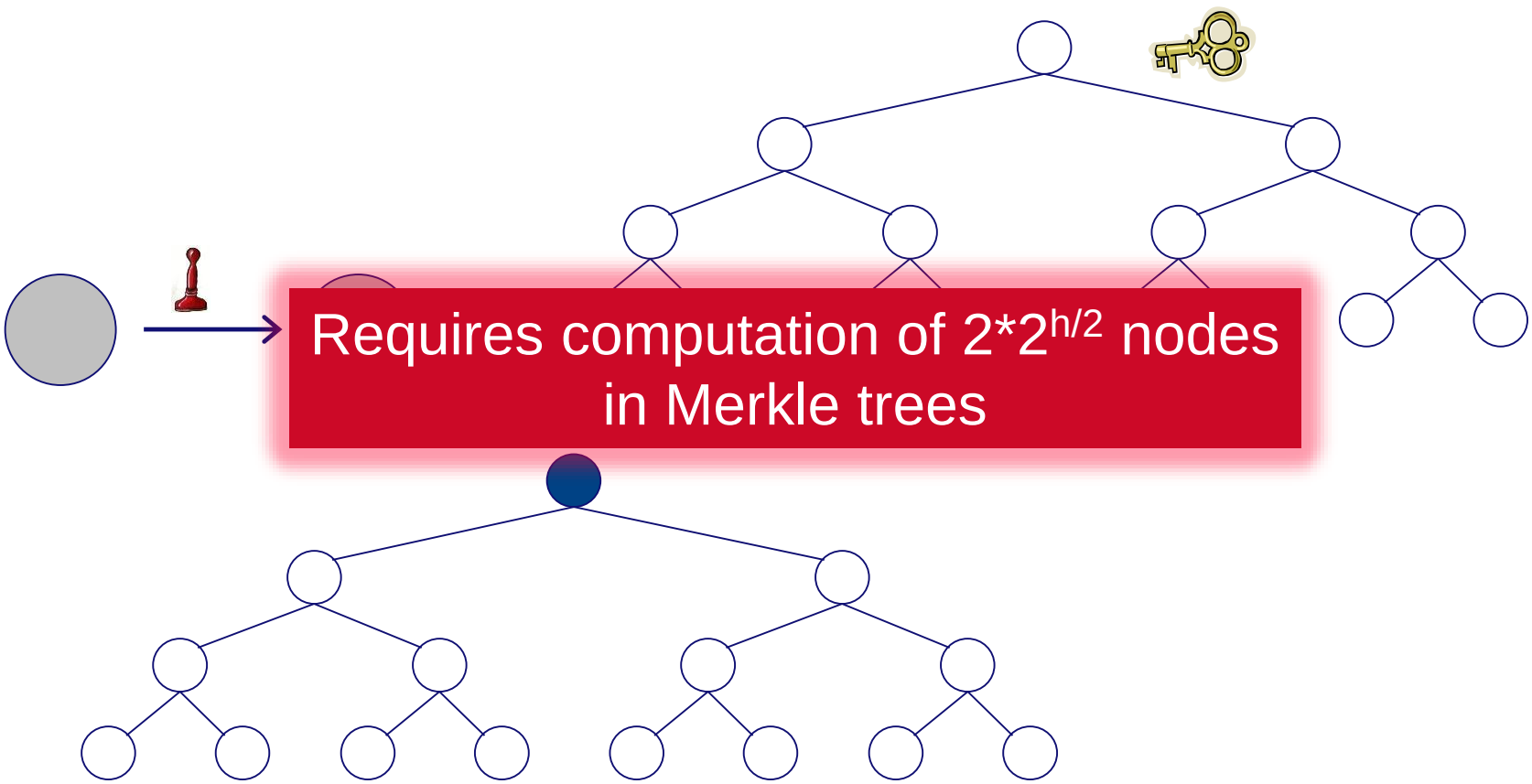
XMSS Public Key Generation

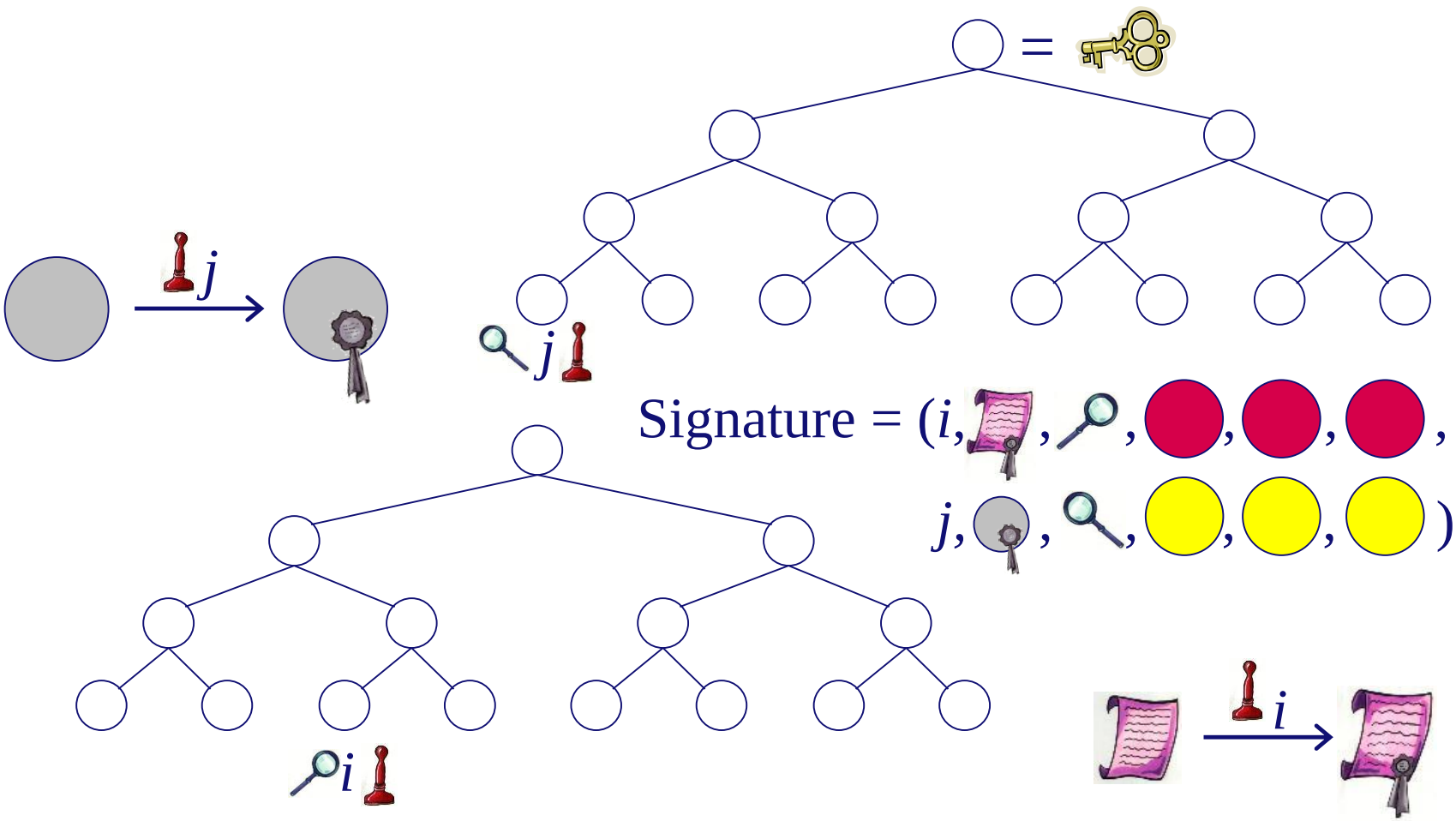


Requires computation of 2^h nodes in Merkle tree

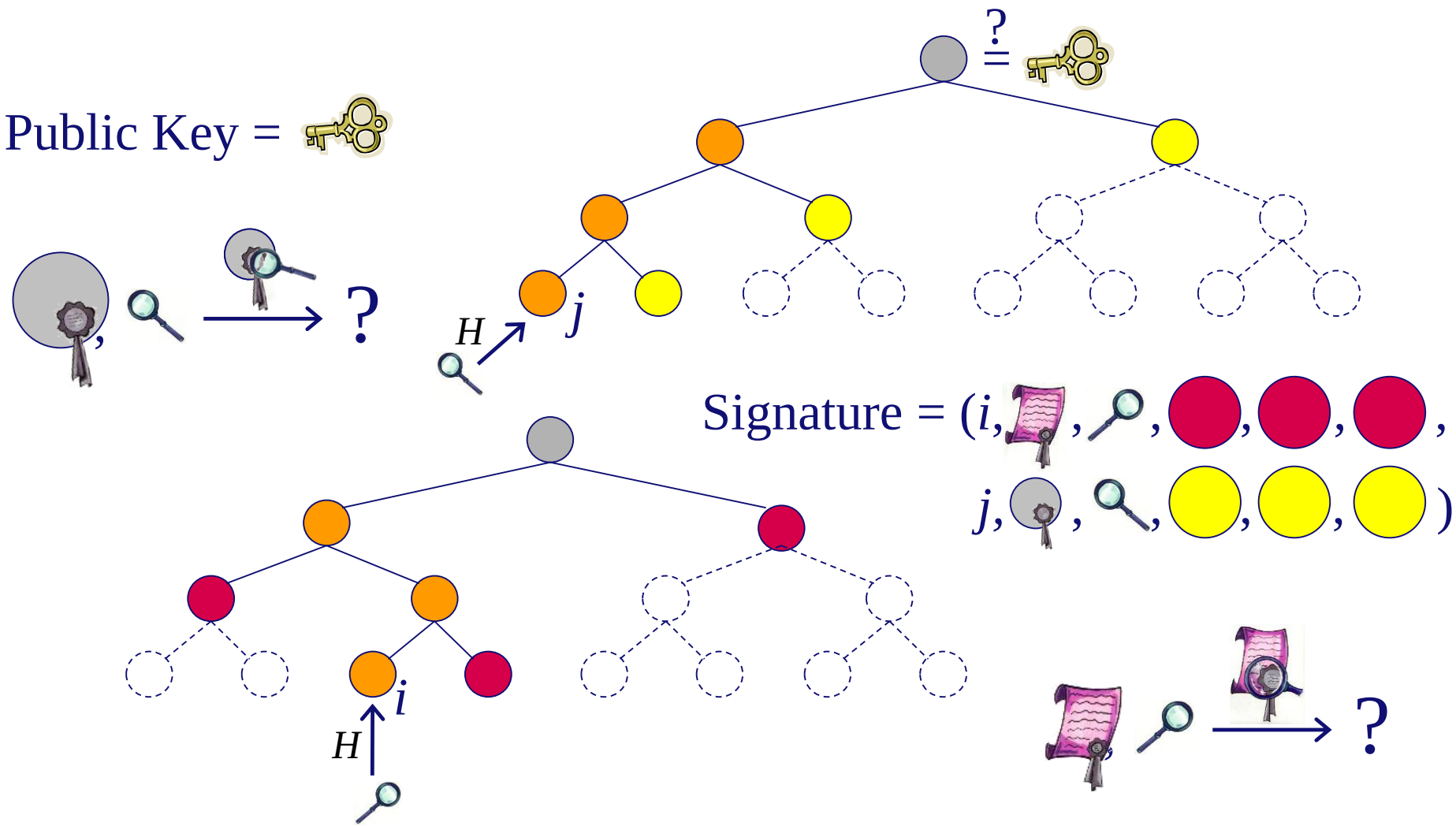
Two Layer
Key generation



Two Layer
Signing



Two Layer
Verifying



XMSS Public Key Generation

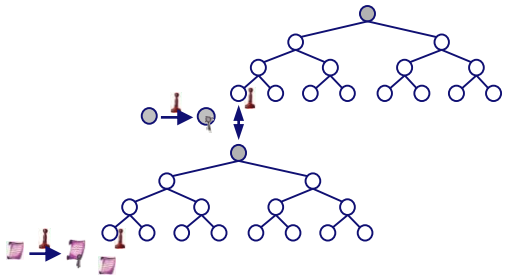
Security level b, tree height h

MSS

- **Generate tree of size 2^h**
- **Cost ~ 2^h**

XMSS

- **Tree chaining**
- **Use d layers of trees of height h/t**
- **Generate d trees of height $2^{h/d}$**
- **Cost ~ $d*2^{h/d}$**
- **Example: h = 40, d = 2, costs ~ $2*2^{20} = 2^{21}$**
- **Slightly increased signature size (+d-1 one-time sigs)**



XMSS Authentication Path Generation

Straight forward: 2^h-1 leaf + 2^h-h node computations

BDS Algorithm:

Runtime

$(h-k)/2+1$ leaf and

$3(h-k-1)/2+1$ node computations.

$+(h-k)$ calls to FSPRG for forward secure XMSS in the worst case.

Storage

$$3h+\left\lfloor \frac{h}{2} \right\rfloor -3k-2+2^k \; n \text{ bit nodes}$$

$+2h-2k$ n bit seeds for forward secure XMSS.

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XMSS Implementations
C Implementation

C Implementation, using OpenSSL [BDH2011]

	Sign (ms)	Verify (ms)	Signature (bit)	Public Key (bit)	Secret Key (byte)	Bit Security	Comment
XMSS-SHA-2	35.60	1.98	16,672	13,600	3,364	157	h = 20, w = 64,
XMSS-AES-NI	0.52	0.07	19,616	7,328	1,684	84	h = 20, w = 4
XMSS-AES	1.06	0.11	19,616	7,328	1,684	84	h = 20, w = 4
RSA 2048	3.08	0.09	≤ 2,048	≤ 4,096	≤ 512	87	

Intel(R) Core(TM) i5-2520M CPU @ 2.50GHz with Intel AES-NI

XMSS Implementations
Smartcard Implementation

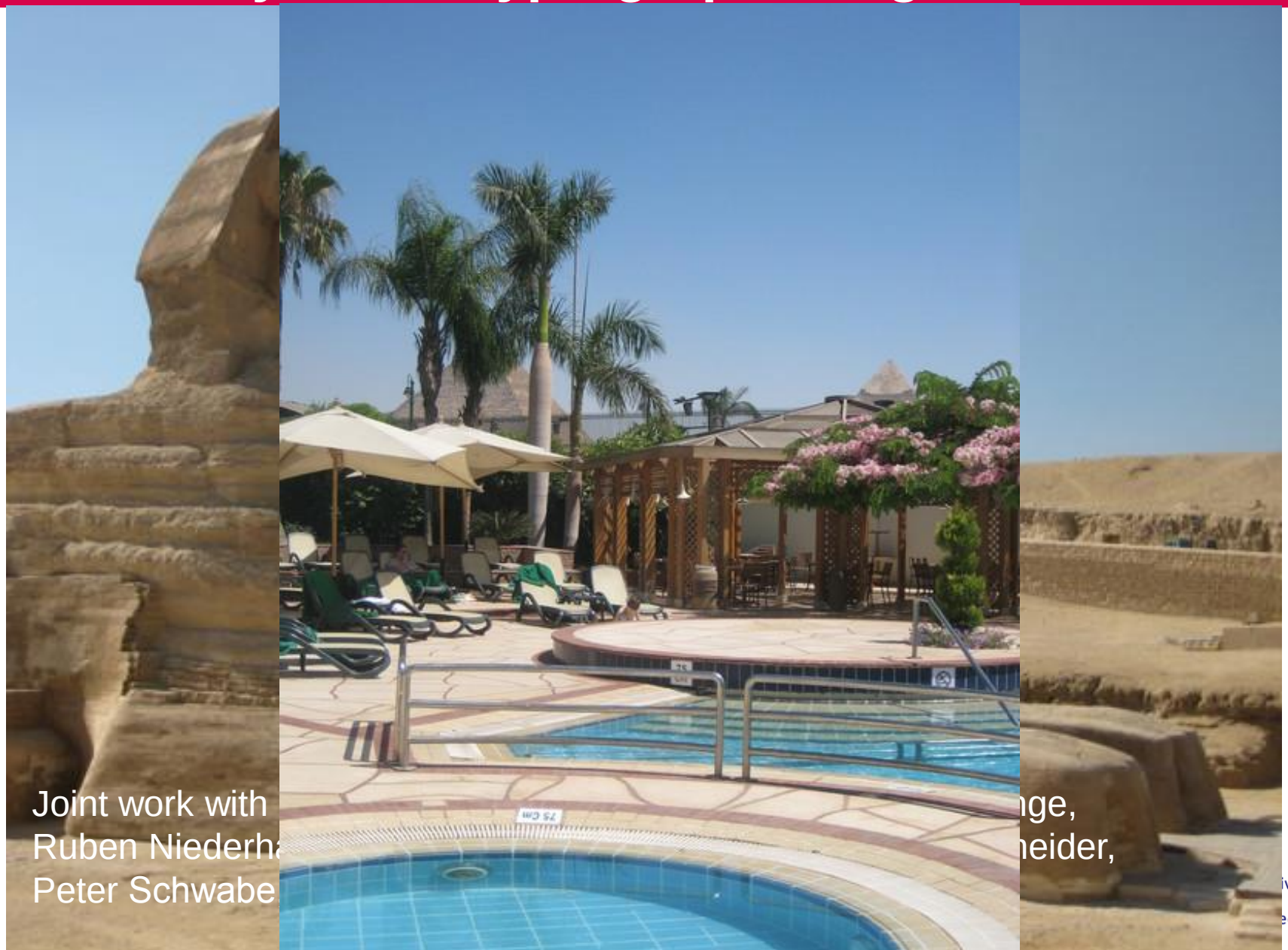
	Sign (ms)	Verify (ms)	Keygen (ms)	Signature (byte)	Public Key (byte)	Secret Key (byte)	Bit Sec.	Comment
XMSS	134	23	925,400	2,388	800	2,448	92	H = 16, w = 4
XMSS$^+$	106	25	5,600	3,476	544	3,760	94	H = 16, w = 4
RSA 2048	190	7	11,000	≤ 256	≤ 512	≤ 512	87	

Infineon SLE78 16Bit-CPU@33MHz, 8KB RAM, TRNG, sym. & asym. co-processor

NVM: Card 16.5 million write cycles/ sector,
XMSS$^+$ < 5 million write cycles (h=20)

[HBB12]

SPHINCS: Stateless Practical Hash-based Incredibly Nice Cryptographic Signatures



Joint work with
Ruben Niederha
Peter Schwabe

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echnology

Long-Standing Problem: Statefulness

- No problem in many cases.
  - Qualified signatures,
  - Keys on smartcard, ...
- Necessary for forward-security!

But:

- Key back-ups undermine security
- Parallel use of key problematic
  - Multi-threading,
  - Load balancing...
- Do not fit standard API

SPHINCS Properties

Stateless

128bit Quantum Security

Practical Speed

Practical Signature Size

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How to Eliminate the State

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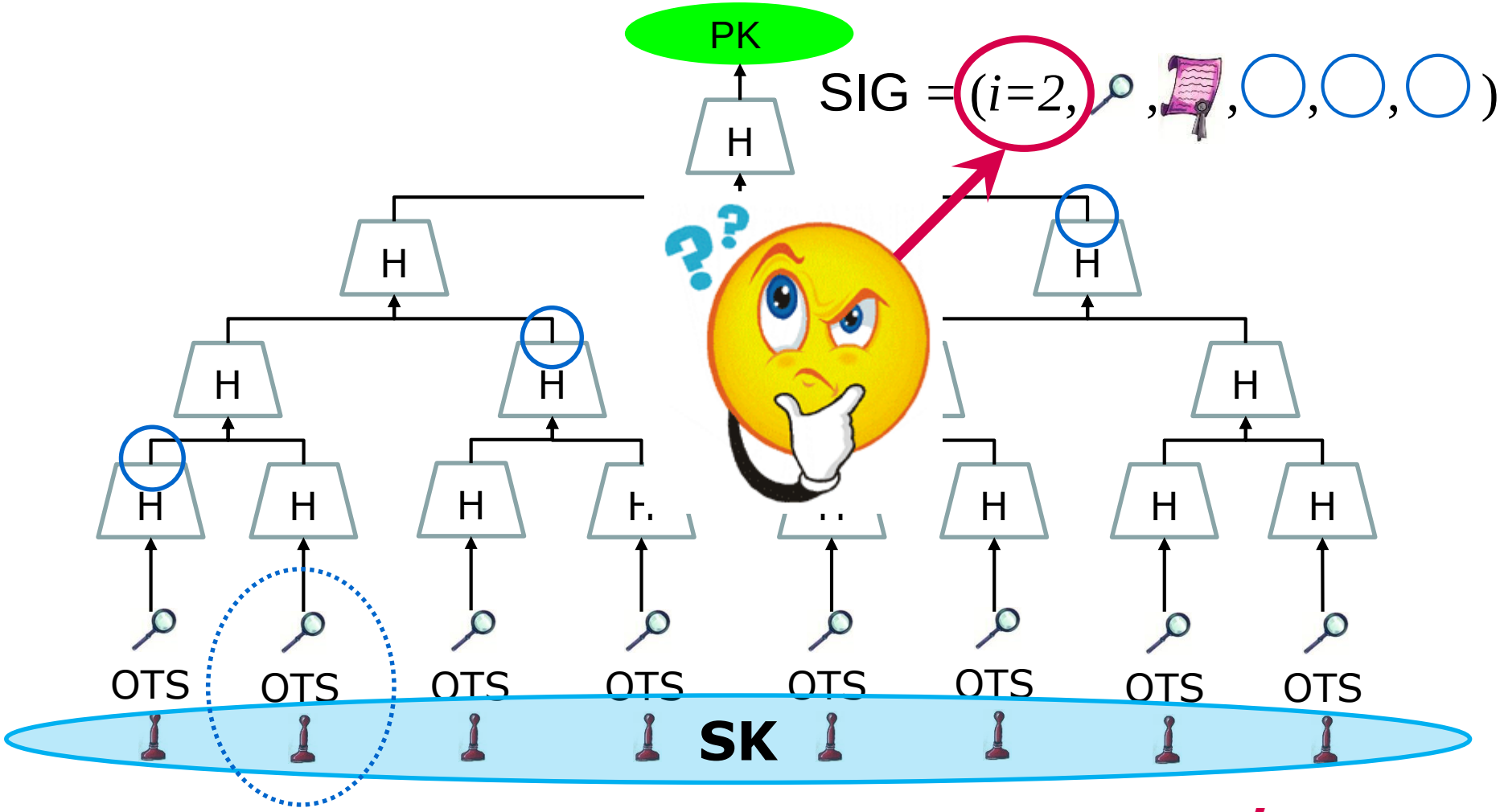
Protest?

Straight Forward

- **Run MSS without State**



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Approach 1: Message Hash

i = Hash(Message);

- 128bit Quantum Sec.

  → $n = 256$ bit Hash [Ber09]

  → #Indices = 2^{256}

  → $h = n = 256$

- h depends on n!
- Best we can do:

  $t_{Sign} \approx n^3 / \log n \; t_{Hash} = 2M \; t_{Hash} \approx 15$ min*

  $|Sig| \approx n^3 / \log n > 256$ kb

* (OpenSSL SHA2)

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Approach 2: Random Index

$$I \leftarrow_{\$} U_{\#Indices}$$

- 128bit Quantum Sec.
  → Sampled by Signer
    → #Indices ← collision prob.
    → #Indices = 2^{256}
    → $h = 256$

- Impossible to make this efficient, again…
- BUT:
  - h independent of n
  - Statistical collision probability NOT collision resistance

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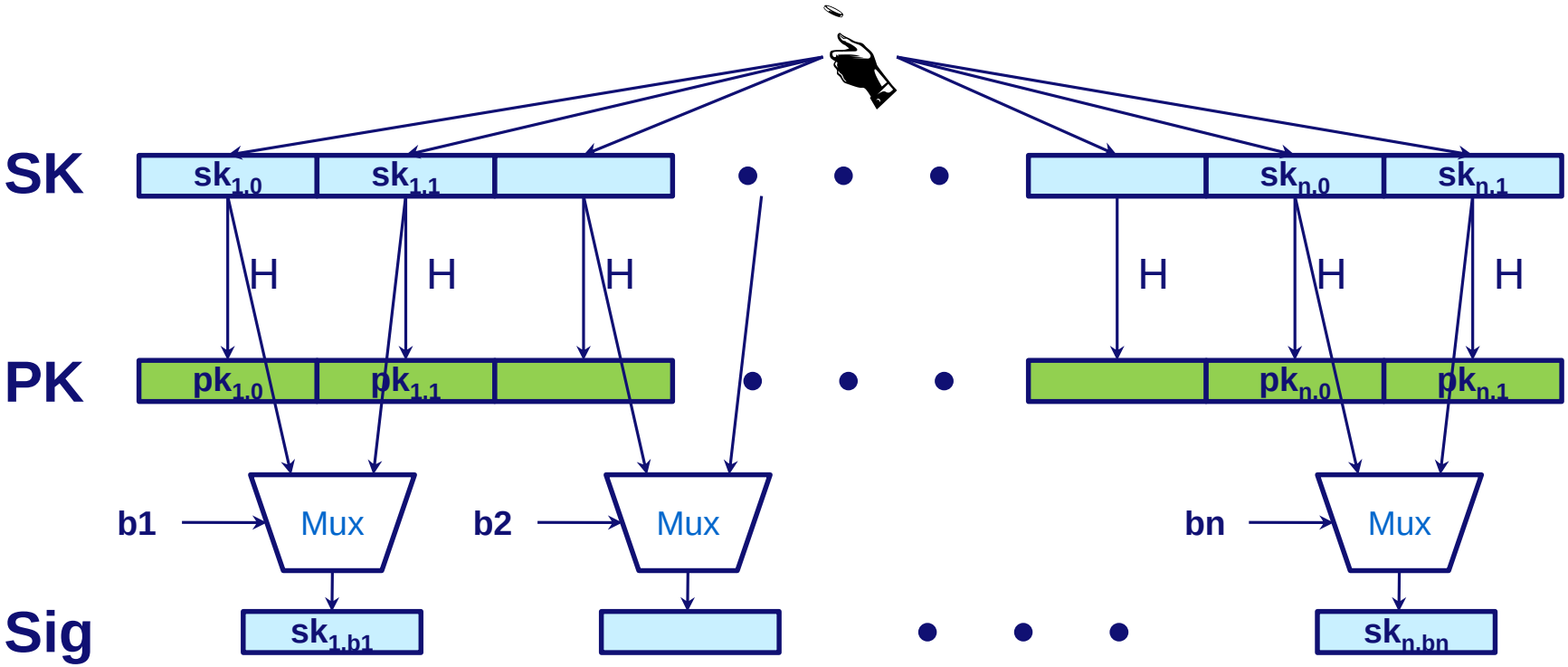
Few-Time Signature Schemes

Recap LD-OTS

Message M = b1,…,bn, OWF H ***** **= n bit**



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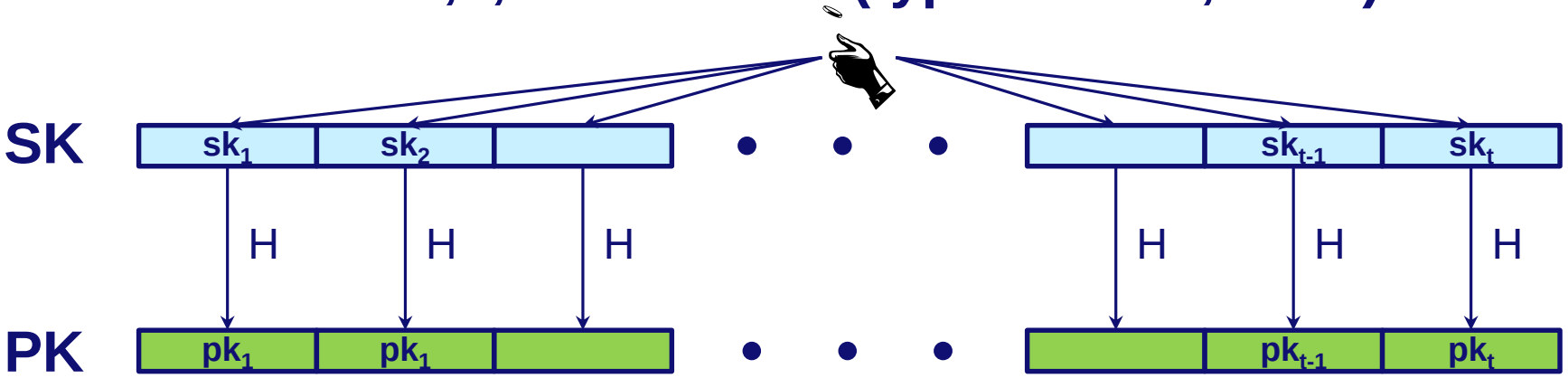
HORS [RR02]

Message M, OWF H, CRHF H' [*] **= n bit**

Parameters $t=2^a$,k, with m = ka (typical a=16, k=32)



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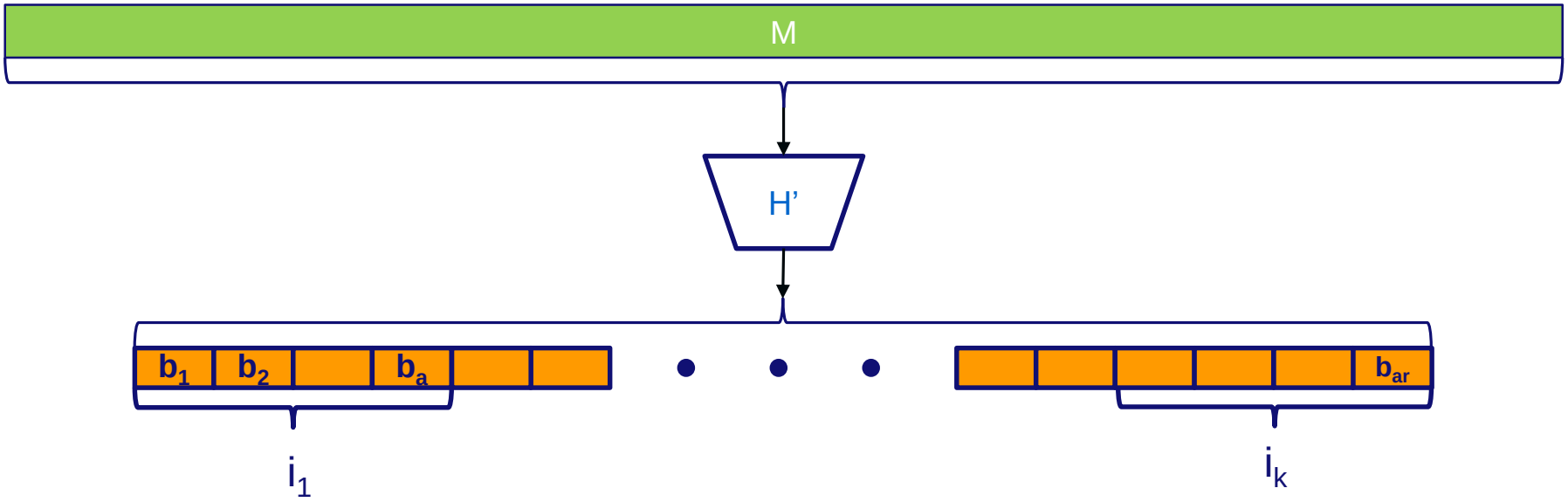
HORS mapping function

Message M, OWF H, CRHF H’ ***** **= n bit**

Parameters $t=2^a$,k, with m = ka (typical a=16, k=32)



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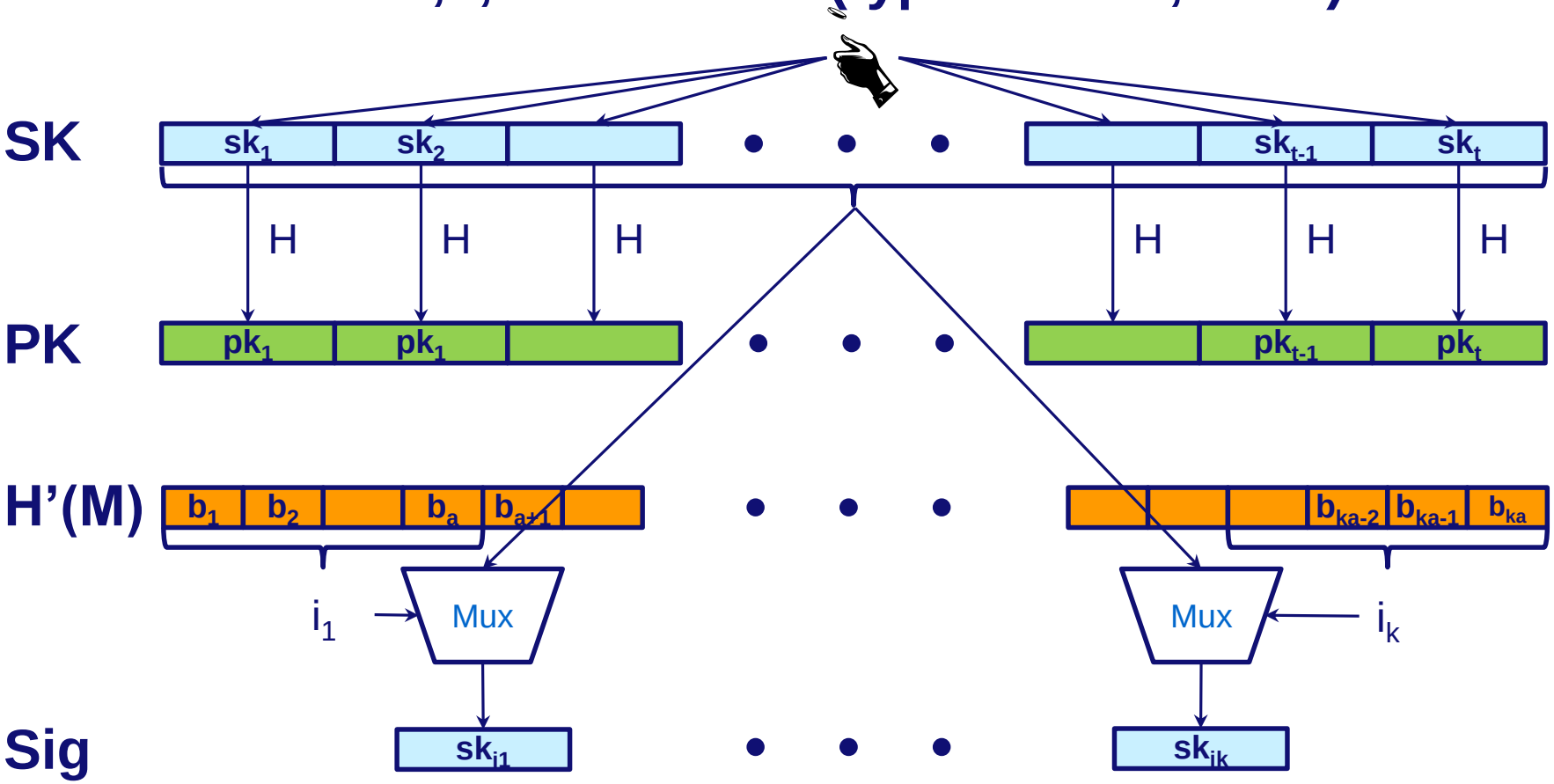
HORS

Message M, OWF H, CRHF H' *** = n bit**

Parameters $t=2^a$,k, with m = ka (typical a=16, k=32)



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HORS Security

- M mapped to k element index set $M^i \in \{1,..,t\}^k$
- Each signature publishes k out of t secrets
- Either break one-wayness or…

- r-Subset-Resilience: After seeing index sets M^i_j for r messages msg_j, $1 <= j <= r$, hard to find $msg_{r+1} \neq msg_j$ such that $M^i_{r+1} \in \cup_{1<=j<=r} M^i_j$.

- Best generic attack: $Succ_{r\text{-}SSR}(A,q) = q(rk / t)^k$

→ Security shrinks with each signature!

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HORST

Using HORS with MSS requires adding PK (tn) to MSS signature.

HORST: Merkle Tree on top of HORS-PK

- **New PK = Root**
- **Publish Authentication Paths for HORS signature values**
- **PK can be computed from Sig**
- **With optimizations: $tn \rightarrow (k(\log t - x + 1) + 2^x)n$**
  - **E.g. SPHINCS-256: 2 MB → 16 KB**
- **Use randomized message hash**

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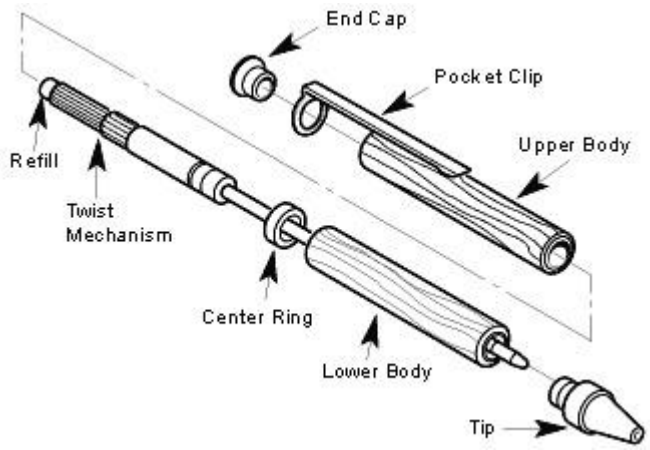
Assembling SPHINCS



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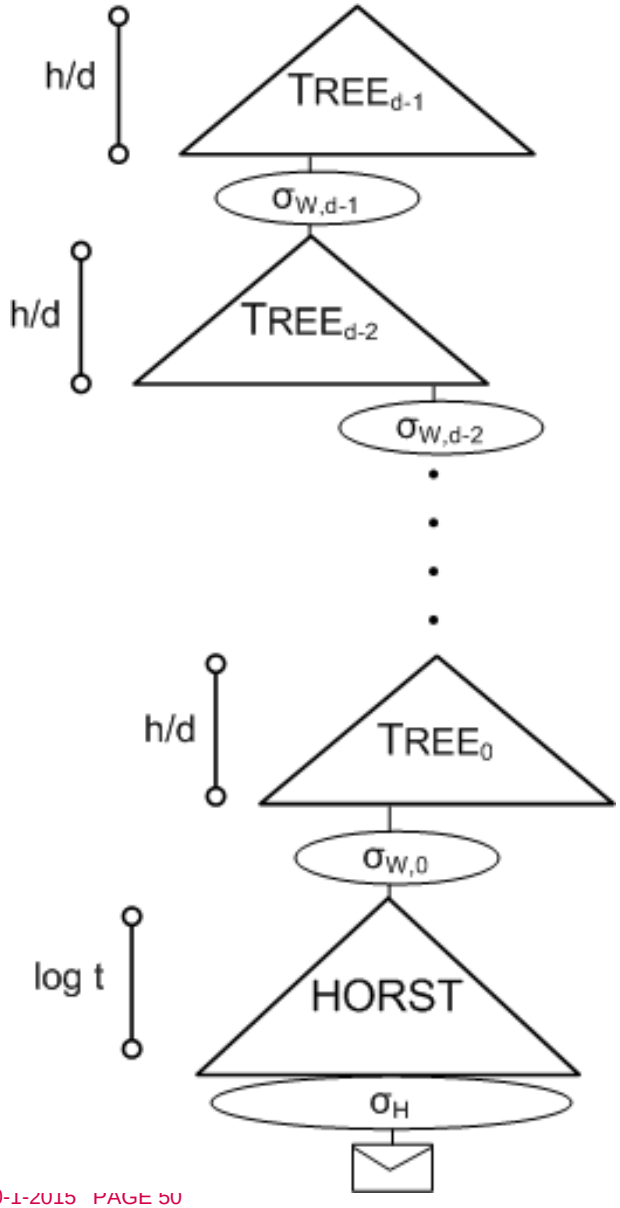
SPHINCS Signature



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SPHINCS Key Ideas

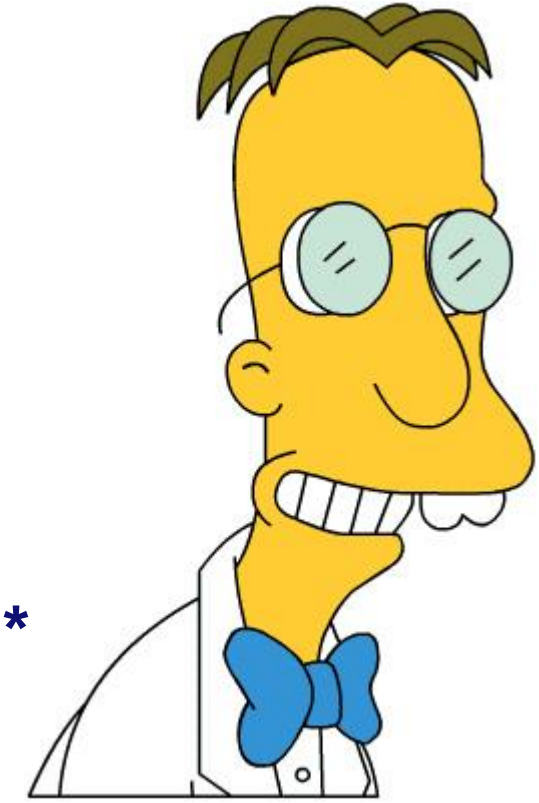
Use HORST key pairs to sign messages

Authenticate HORST key pairs using hypertree (of XMSS trees)

Use random index

Select Parameters such that

$$\text{sum}_{r \in [0,\infty)}(\Pr[\text{r-times index collision}] * \text{Succ}_{\text{r-SSR}}(A)) = \text{negl}(n)$$



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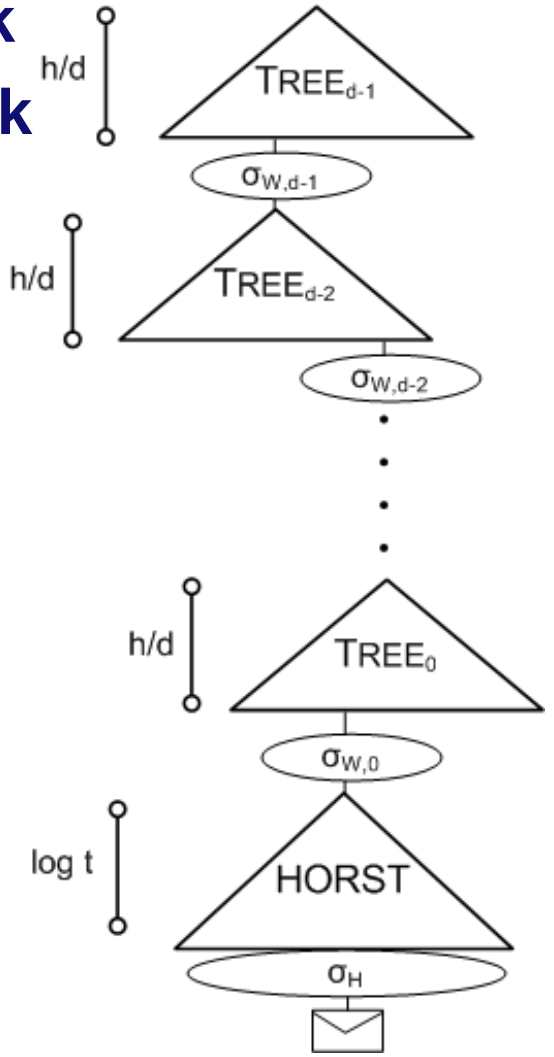
SPHINCS Sign

1. Select (pseudo-)random HORST sk
2. Sign message using this HORST sk
3. Build parent tree
4. Use tree to sign HORST pk
5. If tree != top, goto 3.
6. Output Sig:
   1. Index
   2. HORST signature
   3. XMSS signature chain



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SPHINCS-256

Table 1: SPHINCS-256 parameters and functions for the 128-bit post-quantum security level and resulting signature and key sizes.

Parameter	Value	Meaning
n	256	bitlength of hashes in HORST and WOTS
m	512	bitlength of the message hash
h	60	height of the hyper tree
d	12	layers of the hyper tree
w	16	Winternitz parameter used for WOTS signatures
t	2^{16}	number of secret-key elements of HORST
k	32	number of revealed secret-key elements per HORST signature
Functions		
Hash $\mathcal{H}$:	$\mathcal{H}(R, M) = \text{BLAKE-512}(R\|\|M)$	
PRF $\mathcal{F}_a$:	$\mathcal{F}_a(A, K) = \text{BLAKE-256}(K\|\|A)$	
PRF $\mathcal{F}$:	$\mathcal{F}(M, K) = \text{BLAKE-512}(K\|\|M)$	
PRG G_λ:	$\text{G}_\lambda(\text{SEED}) = \text{ChaCha12}_{\text{SEED}}(0)_{0,\ldots,\lambda-1}$	
Hash F:	$\text{F}(M_1) = \text{CHOP}(\pi_{ChaCha}(M_1\|\|C), 256)$	
Hash H:	$\text{H}(M_1\|\|M_2) = \text{CHOP}\left(\pi_{ChaCha}\left(\pi_{ChaCha}\left(M_1\|\|C\right) \oplus \left(M_2\|\|0^{256}\right)\right), 256\right)$	
Sizes		
Signature size:	41000 bytes	
Public-key size:	1056 bytes	
Private-key size:	1088 bytes	

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SPHINCS-256 Speed

- **Key generation:** 3,051,562 cycles
- **Verification:** 1,369,060 cycles
- **Signature:** 47,466,005 cycles

- **Still hundreds of messages per second on a modern 4-core 3.5GHz Intel CPU (13.56 ms / Sig on 1 Core)**

- **Remember: Optimized Folklore**

$$t_{Sign} \approx 15 \text{ min}^*$$
$$|Sig| > 256 \text{ kb}$$

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In Paper (http://eprint.iacr.org/2014/795)

- Standard model security reduction without collision resistance
- Complexity of generic quantum attacks
- Efficient fixed-input length hashing
- Optimized implementation

Advantages of Hash-based Signatures

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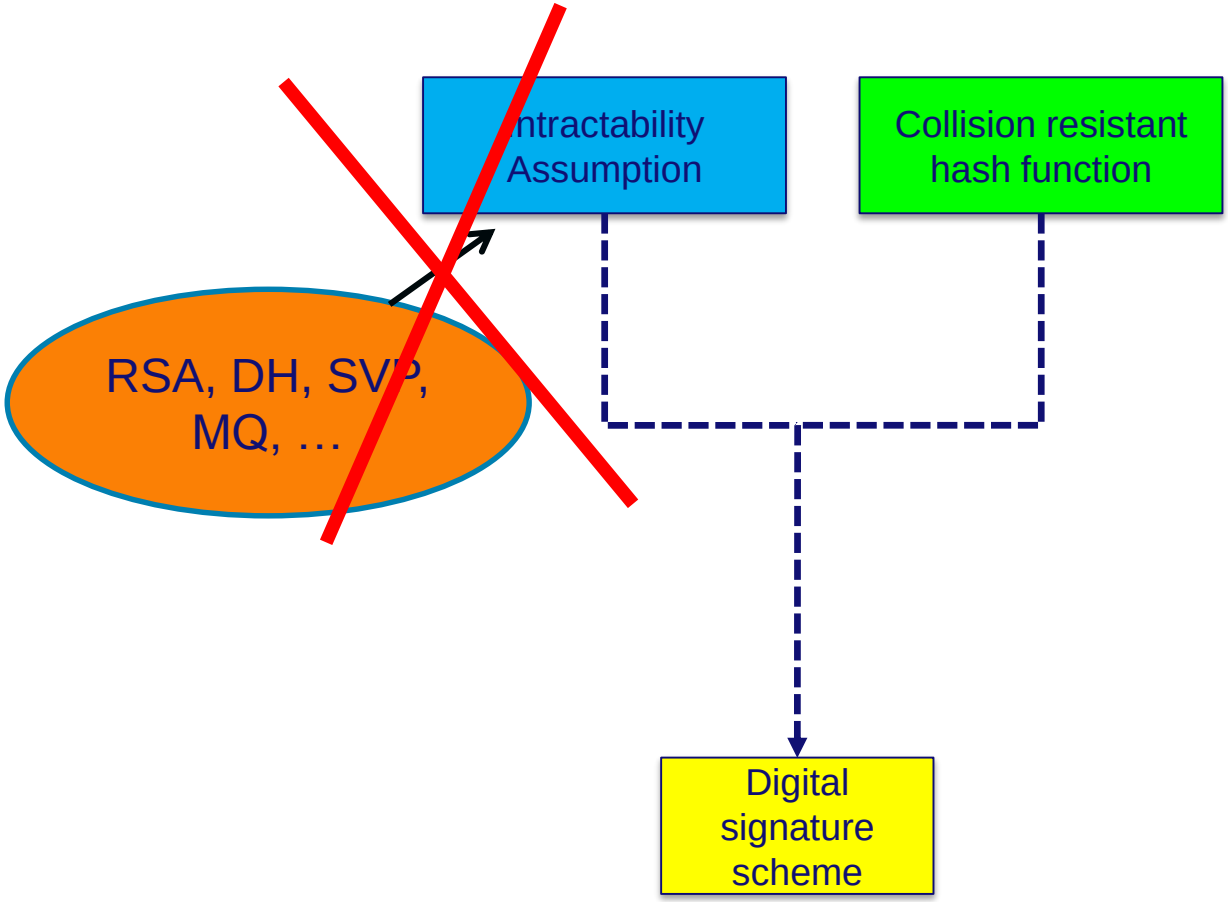
RSA – DSA – EC-DSA...



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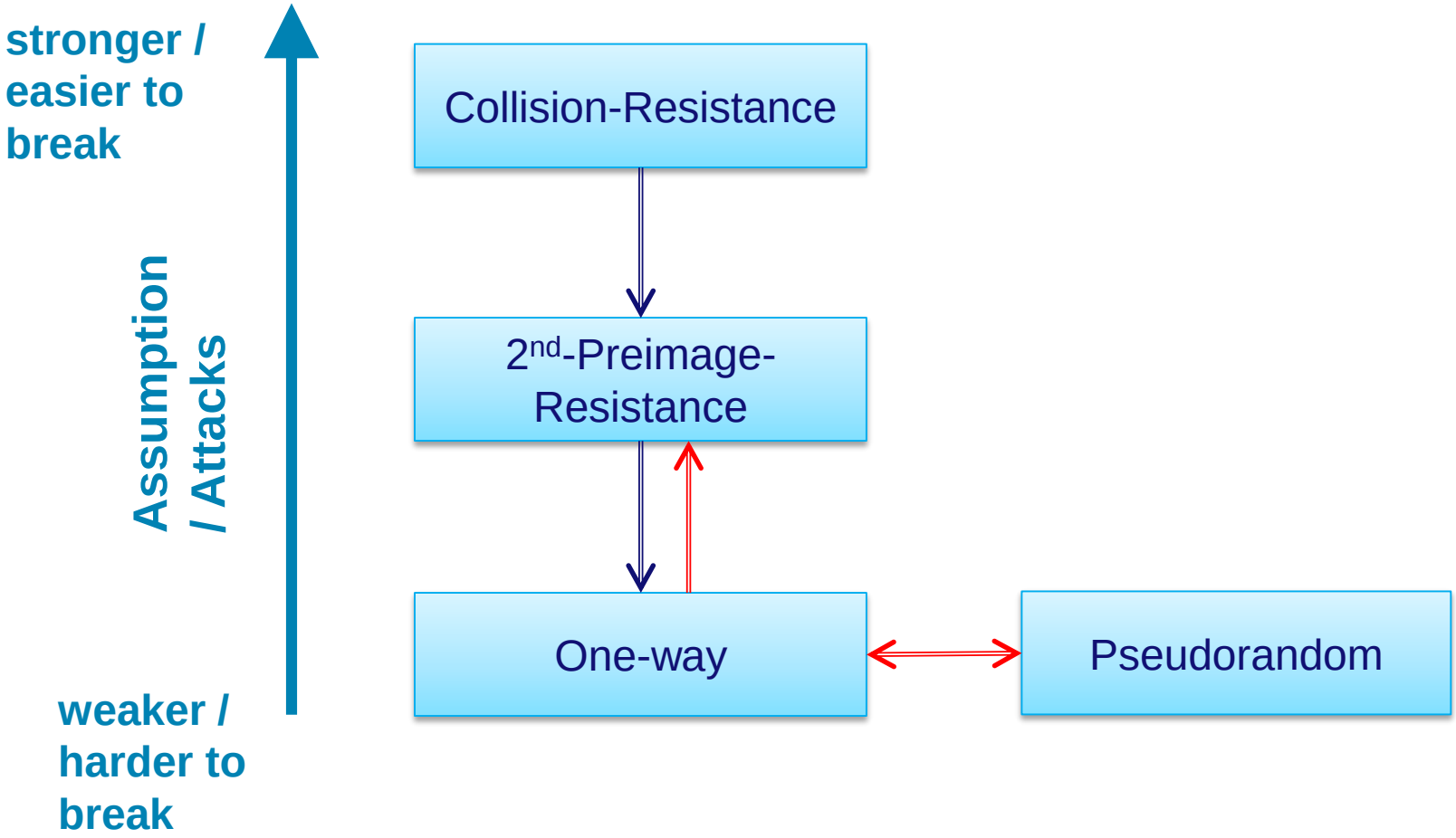
Early Warning System (only XMSS & SPHINCS)
Hash-function properties



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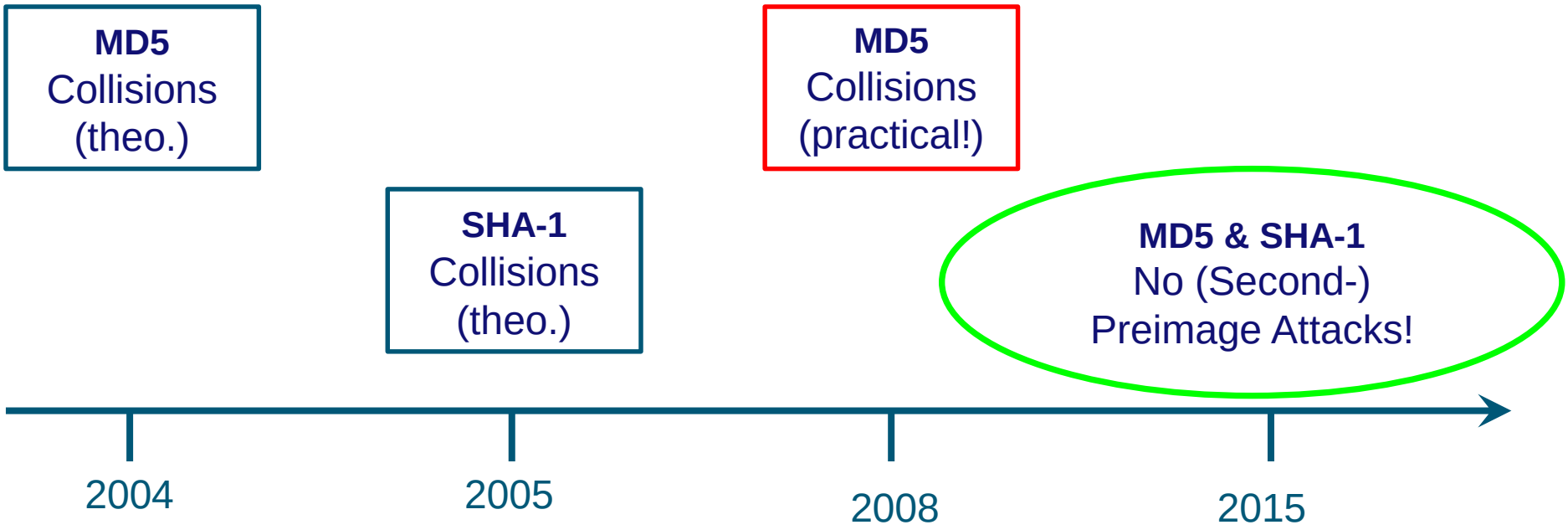
Early Warning System (only XMSS & SPHINCS)

Attacks on Hash Functions



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Easy Redundancy

Hash-Combiner

- Collision-Resistance / 2nd-Preimage-Resistance:

$$h_k(x) = g_k(x) \| f_k(x)$$

- PRF:

$$h_k(x) = g_k(x) \oplus f_k(x)$$

- **No sudden break**
- **Replaces double signature**
- **Signature size only grows by h*n**
- **Runtime ~ doubled**

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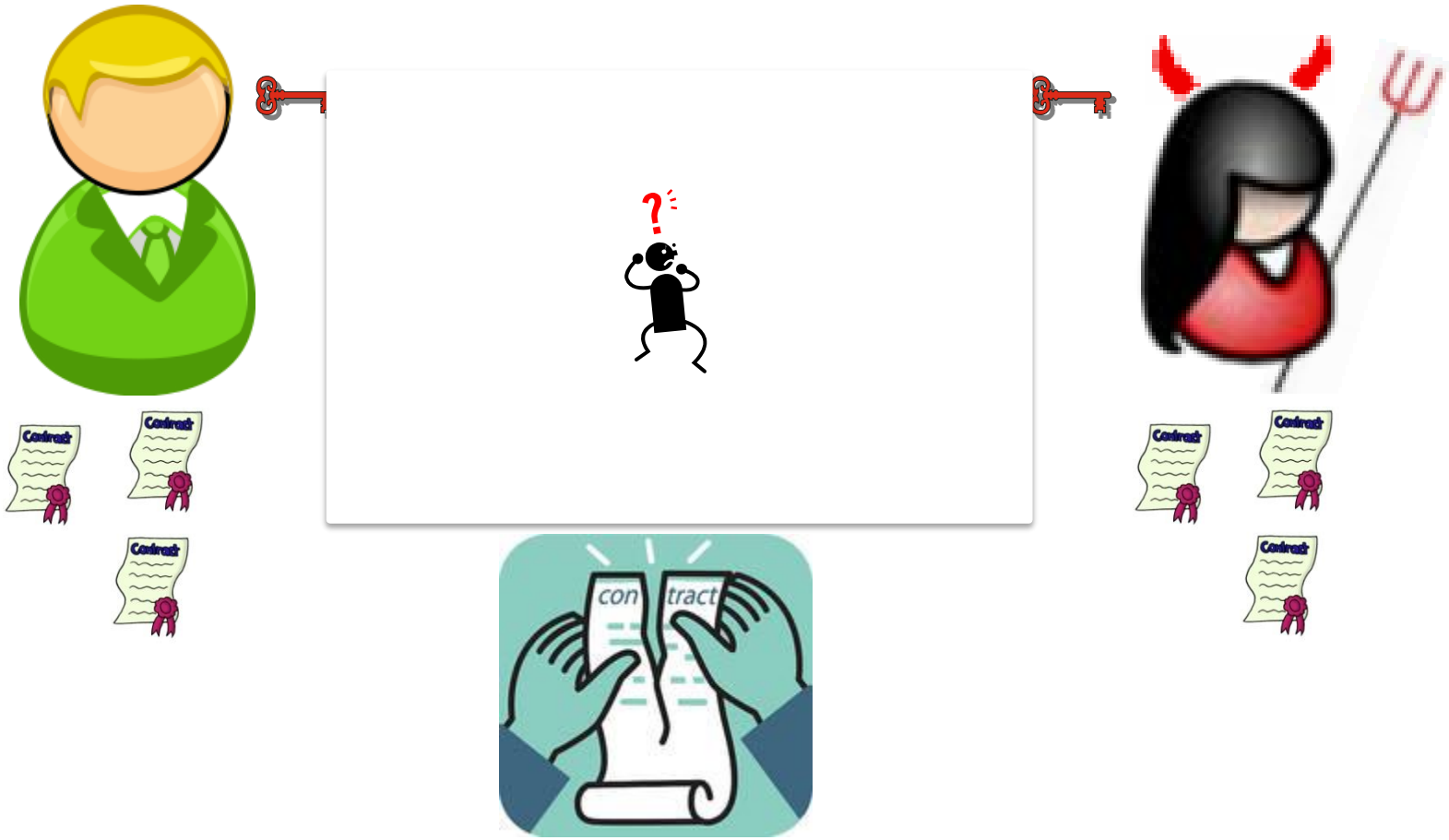
Forward Security (only XMSS)



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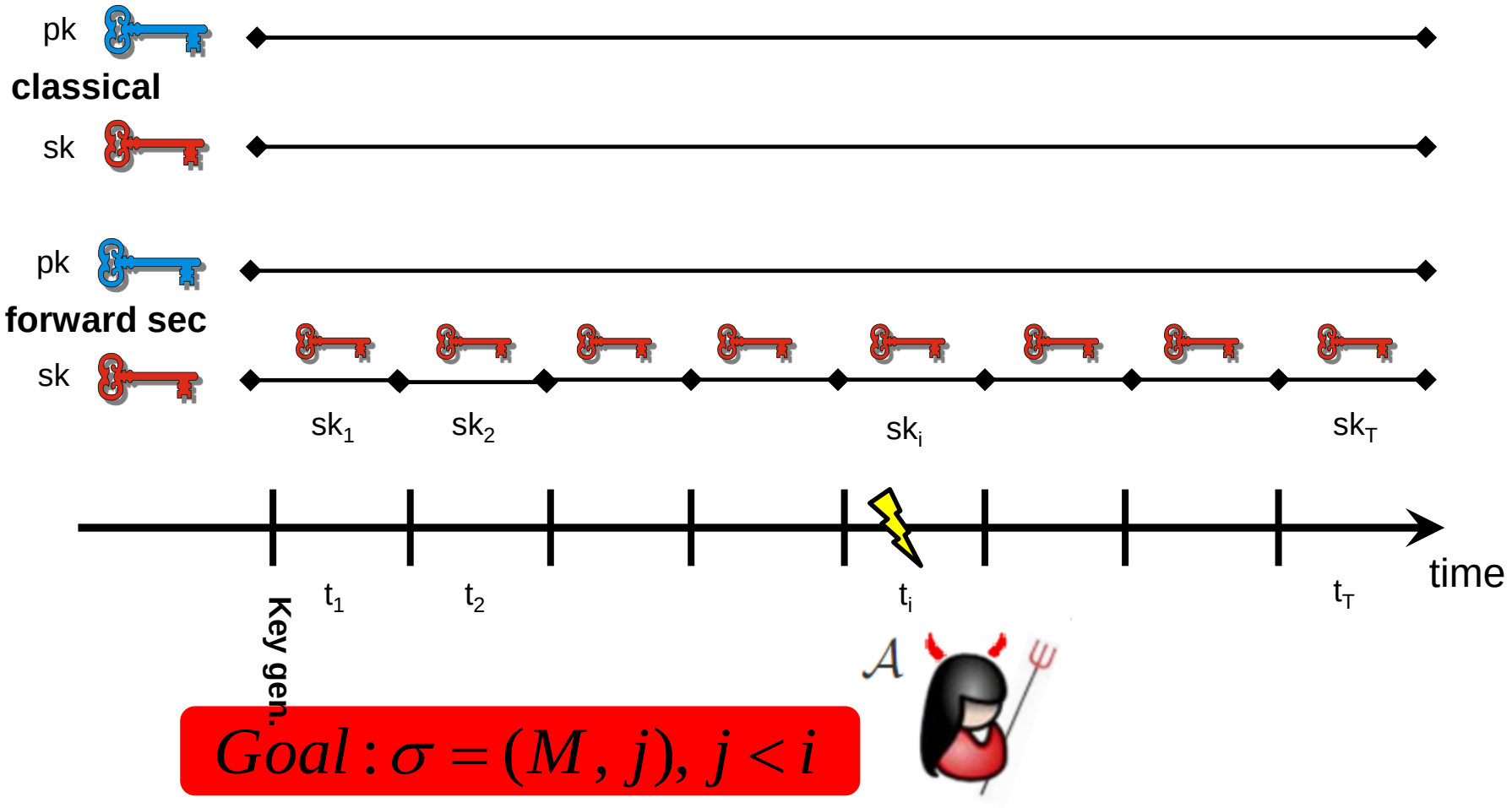
Forward Security - cont'd



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Conclusion

- **Hash-based signatures currently most confidence inspiring pq-signature scheme**
- **If you can live with a state: Go for XMSS.**
- **Otherwise:**
  - **Go for Sphincs-256!**
  - **First stateless signature scheme with post-quantum secure parameters**
  - **Practical speed and sizes**

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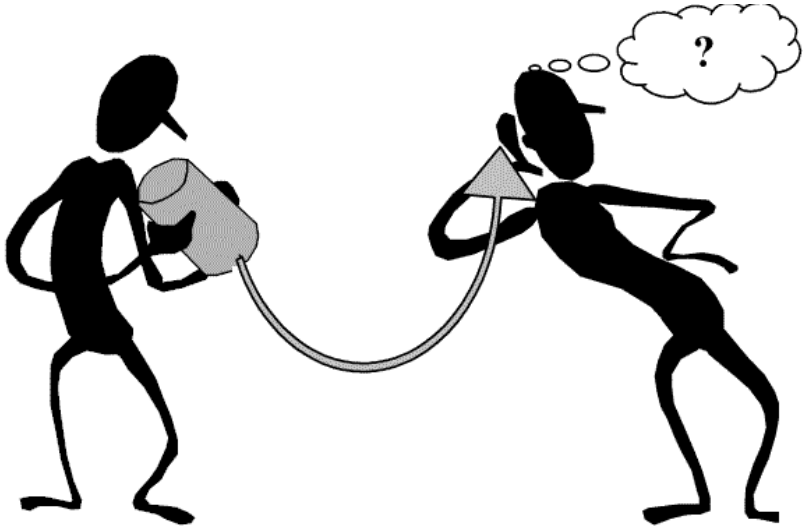
Thank you!

Questions?



For references & further literature see
https://huelsing.wordpress.com/hash-based-signature-schemes/literature/