Hash-based Signatures

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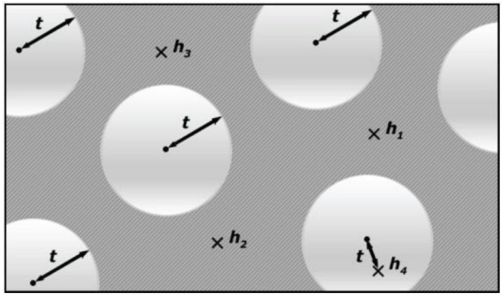
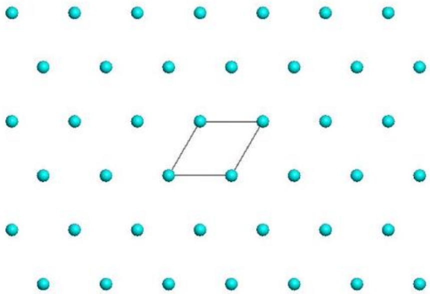
Post-Quantum Signatures

Lattice, MQ, Coding



- Signature and/or key sizes
- Runtimes
- Secure parameters

$$y_1 = x_1^2 + x_1x_2 + x_1x_4 + x_3$$
$$y_2 = x_3^2 + x_2x_3 + x_2x_4 + x_1 + 1$$
$$y_3 = \ldots$$



Hash-based Signature Schemes

[Mer89]

- Post quantum
- Only secure hash function
- Security well understood
- Fast

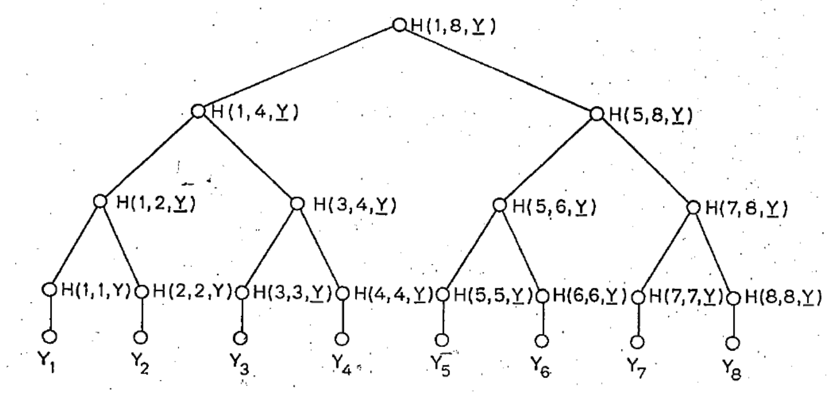


FIG 1
AN AUTHENTICATION TREE WITH N = 8.
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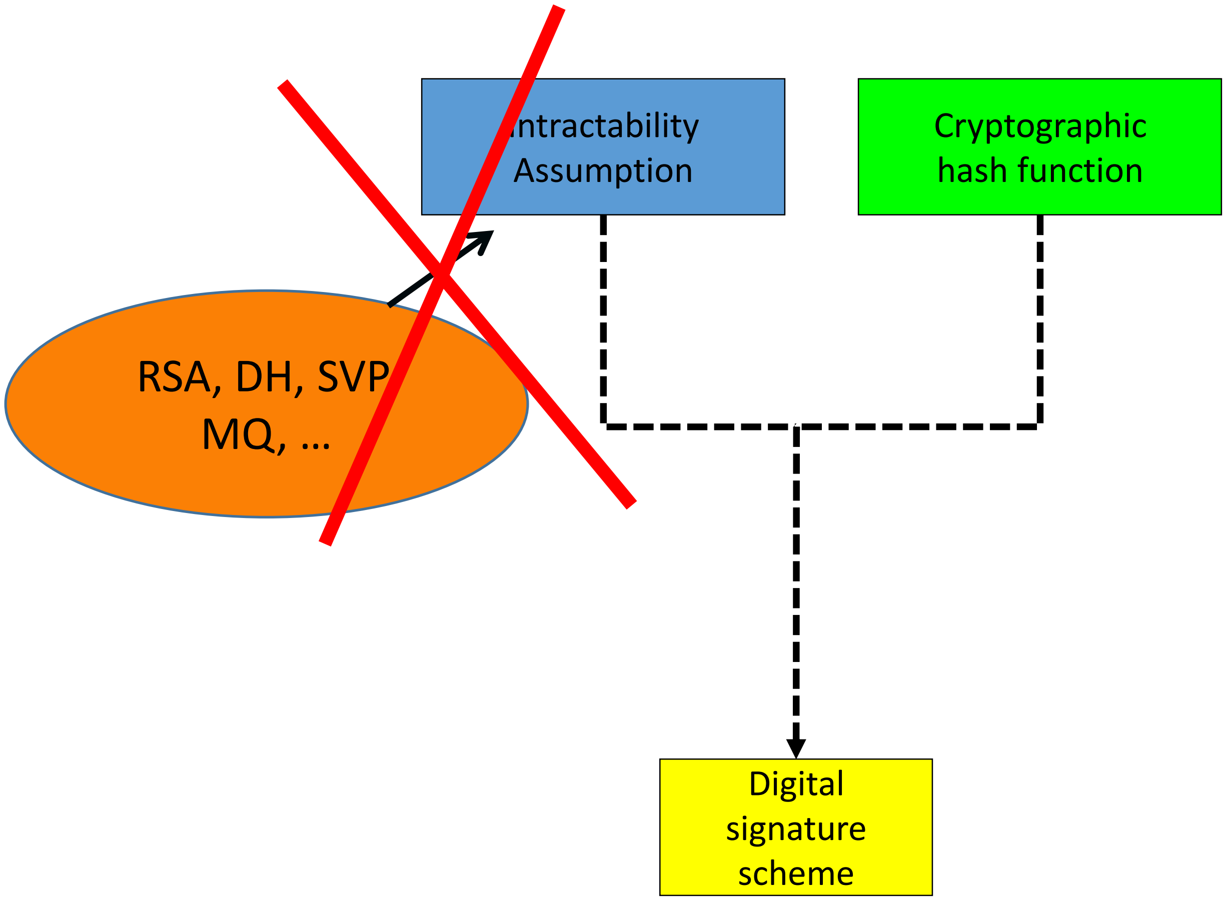
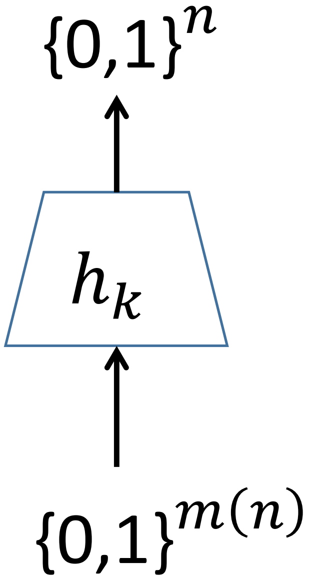
RSA – DSA – EC-DSA…



(Hash) function families

- $H_n := \{h_k : \{0,1\}^{m(n)} \to \{0,1\}^n\}$
- $m(n) \geq n$
- „efficient“



One-wayness

$H_n \coloneqq \left\{h_k\colon \{0,1\}^{m(n)} \rightarrow \{0,1\}^n\right\}$

$h_k \xleftarrow{\$} H_n$

$x \xleftarrow{\$} \{0,1\}^{m(n)}$

$y_c \leftarrow h_k(x)$

Success if $h_k(x^*) = y_c$

y_c, k

$\downarrow$

$\mathcal{A}$

$\downarrow$

x^*

Collision resistance

$H_n \coloneqq \{h_k : \{0,1\}^{m(n)} \to \{0,1\}^n\}$

$h_k \overset{\$}{\leftarrow} H_n$

Success if
$h_k(x_1^*) = h_k(x_2^*)$

k

$\mathcal{A}$

(x_1^*, x_2^*)

Second-preimage resistance

$H_n \coloneqq \{h_k \colon \{0,1\}^{m(n)} \to \{0,1\}^n\}$

$h_k \xleftarrow{\$} H_n$

$x_c \xleftarrow{\$} \{0,1\}^{m(n)}$

Success if

$h_k(x_c) = h_k(x^*)$

x_c, k

$\downarrow$

$\mathcal{A}$

$\downarrow$

x^*

Undetectability

$H_n \coloneqq \{h_k : \{0,1\}^{m(n)} \to \{0,1\}^n\}$

$h_k \xleftarrow{\$} H_n$

$b \xleftarrow{\$} \{0,1\}$

If $b = 1$

$\quad x \xleftarrow{\$} \{0,1\}^{m(n)}$

$\quad y_c \leftarrow h_k(x)$

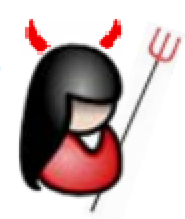
else

$\quad y_c \xleftarrow{\$} \{0,1\}^n$

y_c, k

$\downarrow$

$\mathcal{A}$

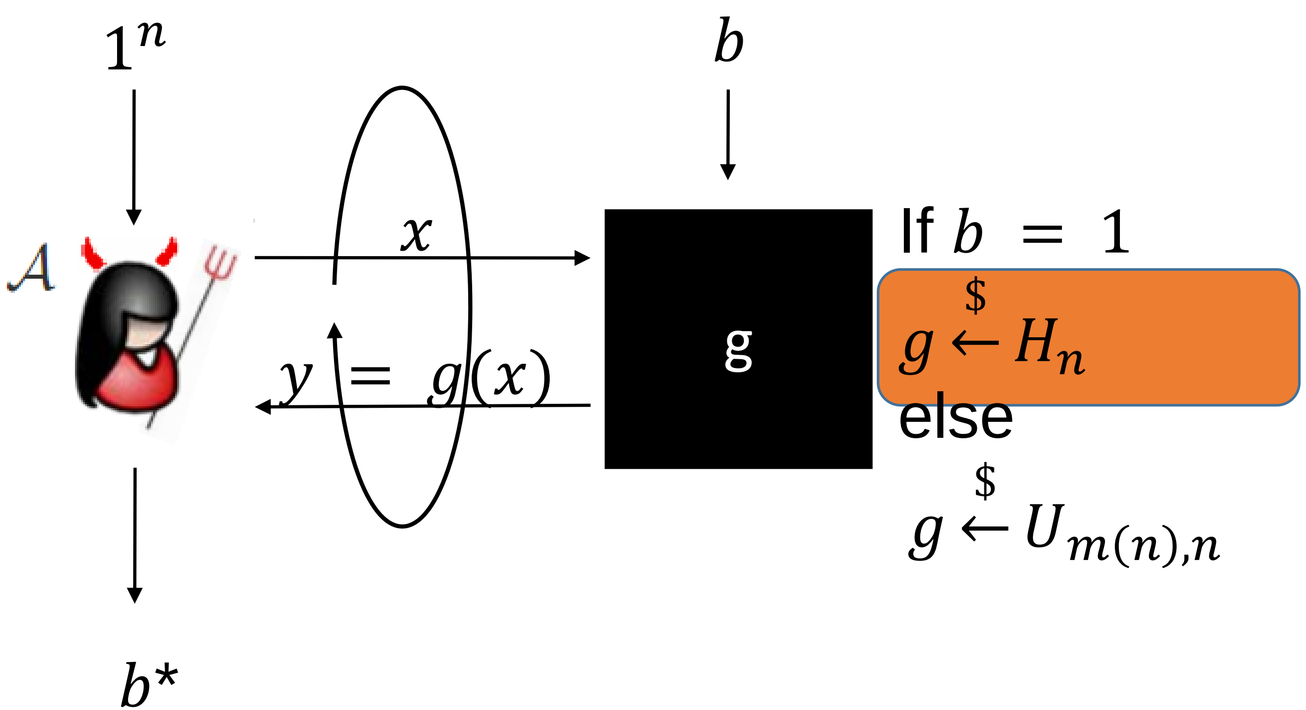


$\downarrow$

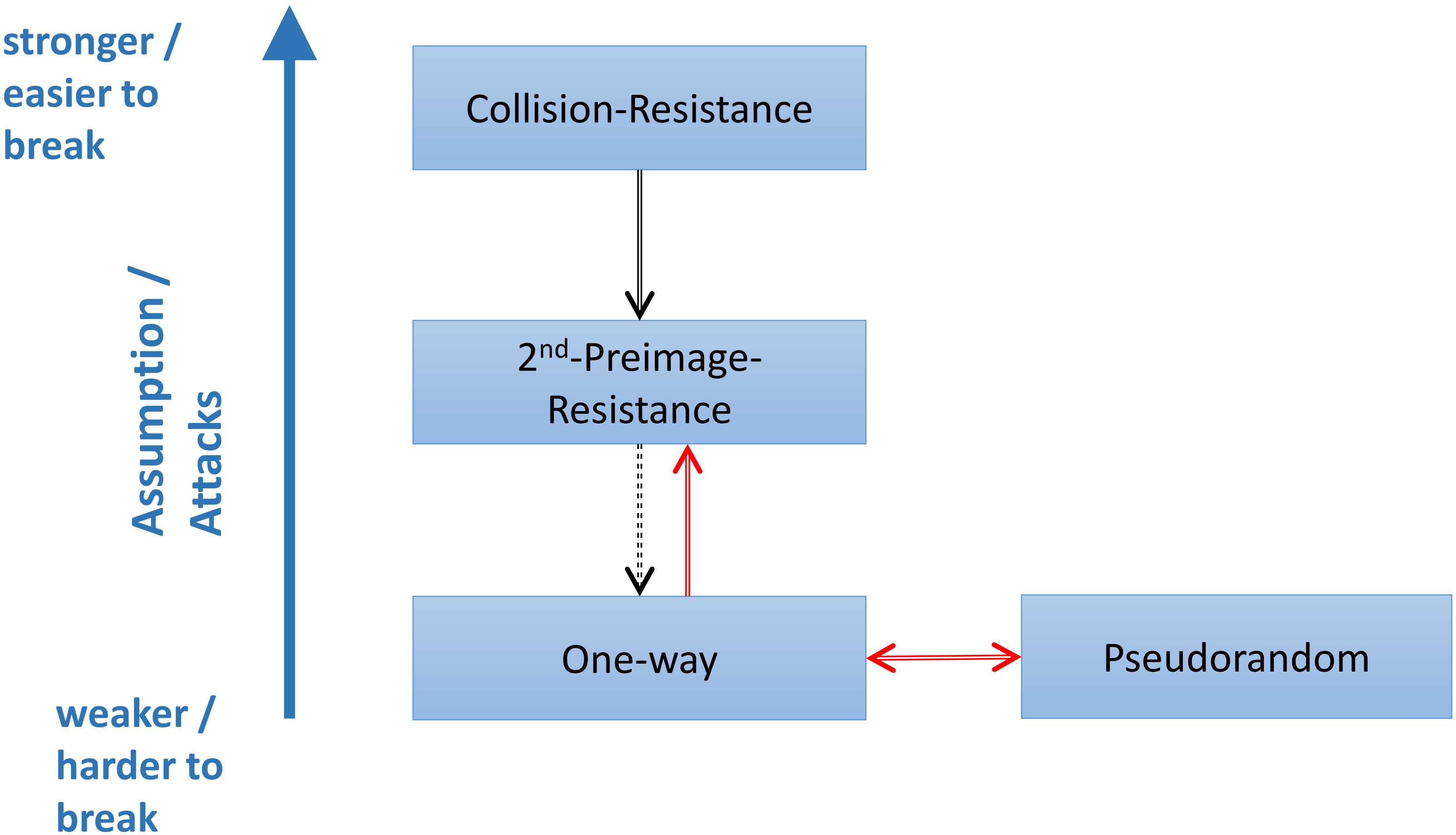
b^*

Pseudorandomness

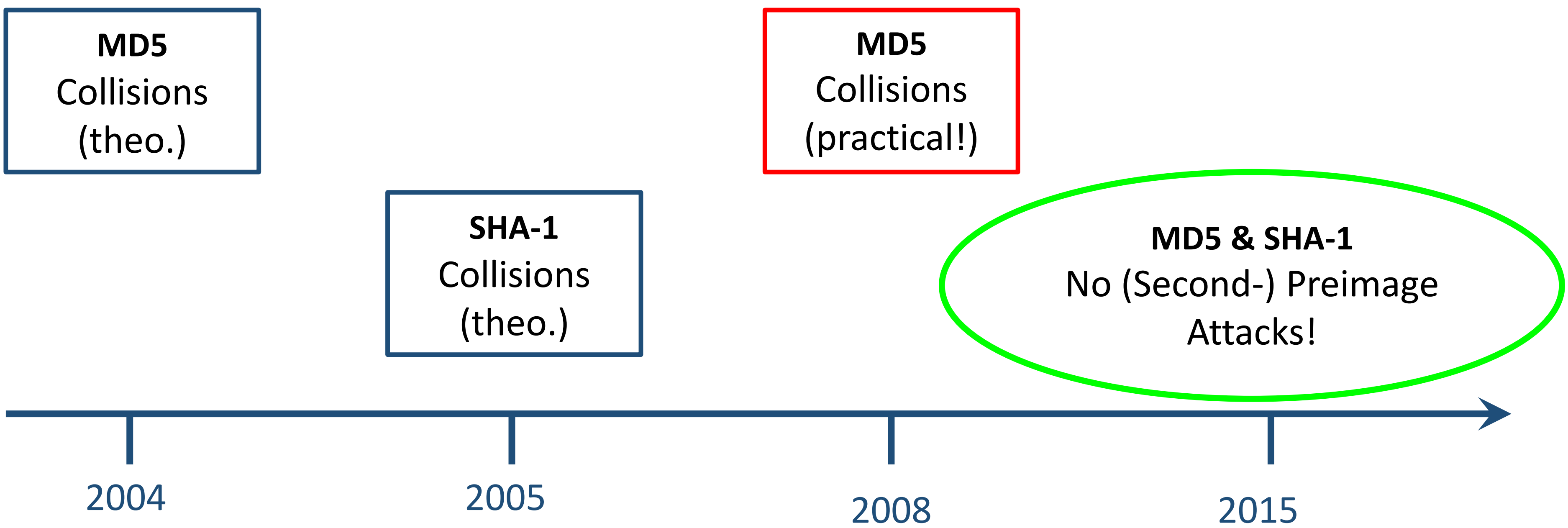
$$H_n := \{h_k : \{0,1\}^{m(n)} \to \{0,1\}^n\}$$



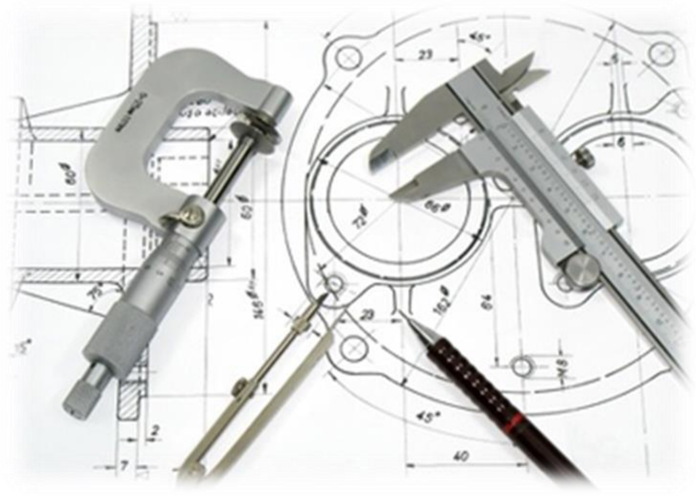
Hash-function properties



Attacks on Hash Functions

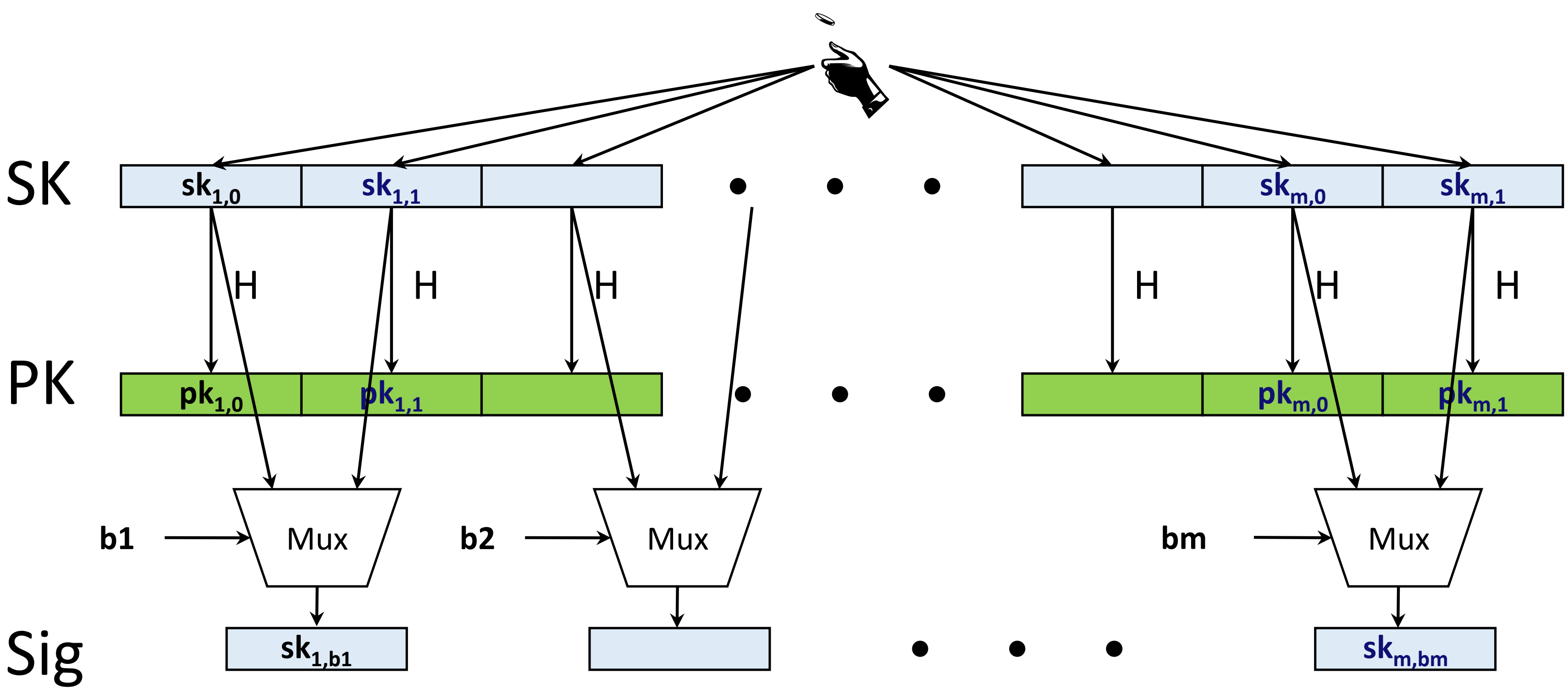


Basic Construction

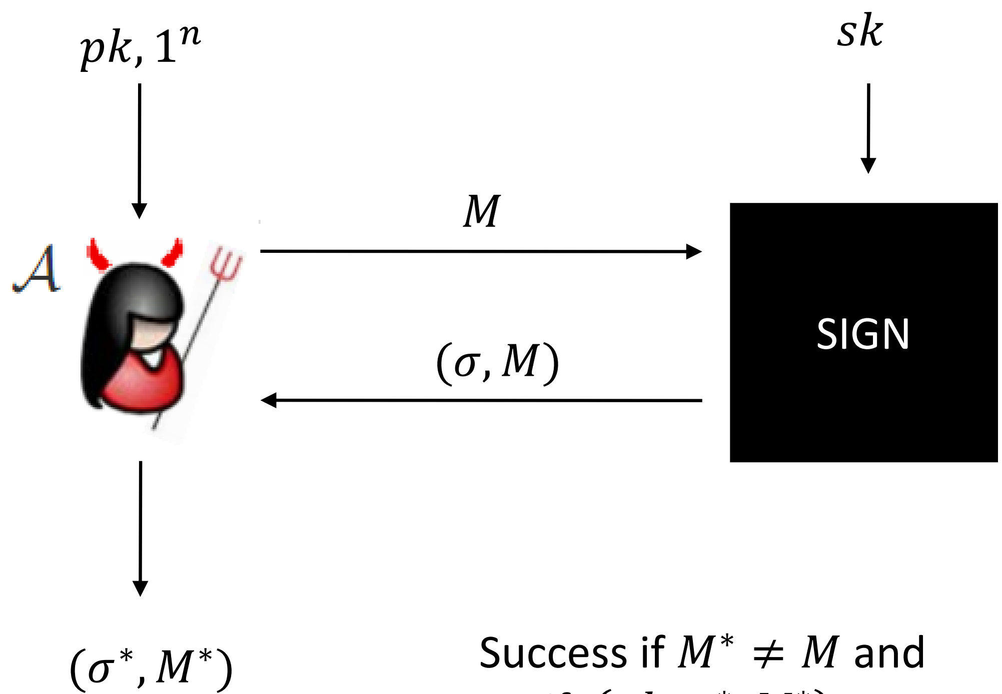


Lamport-Diffie OTS [Lam79]

Message M = b1,…,bm, OWF H   * = n bit



EU-CMA for OTS



Success if $M^* \neq M$ and
Verify$(pk, \sigma^*, M^*) = $ Accept

Security

Theorem:

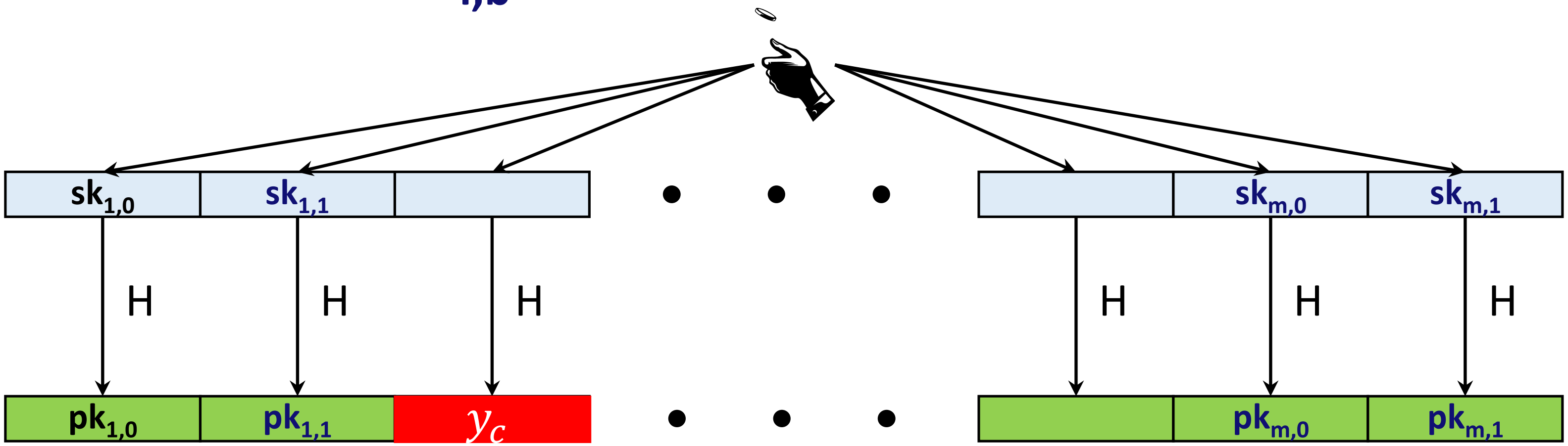
If H is one-way then LD-OTS is one-time eu-cma-secure.

Reduction

Input: y_c, k

Set $H \leftarrow h_k$

Replace random **$pk_{i,b}$**



Reduction

Input: y_c, k

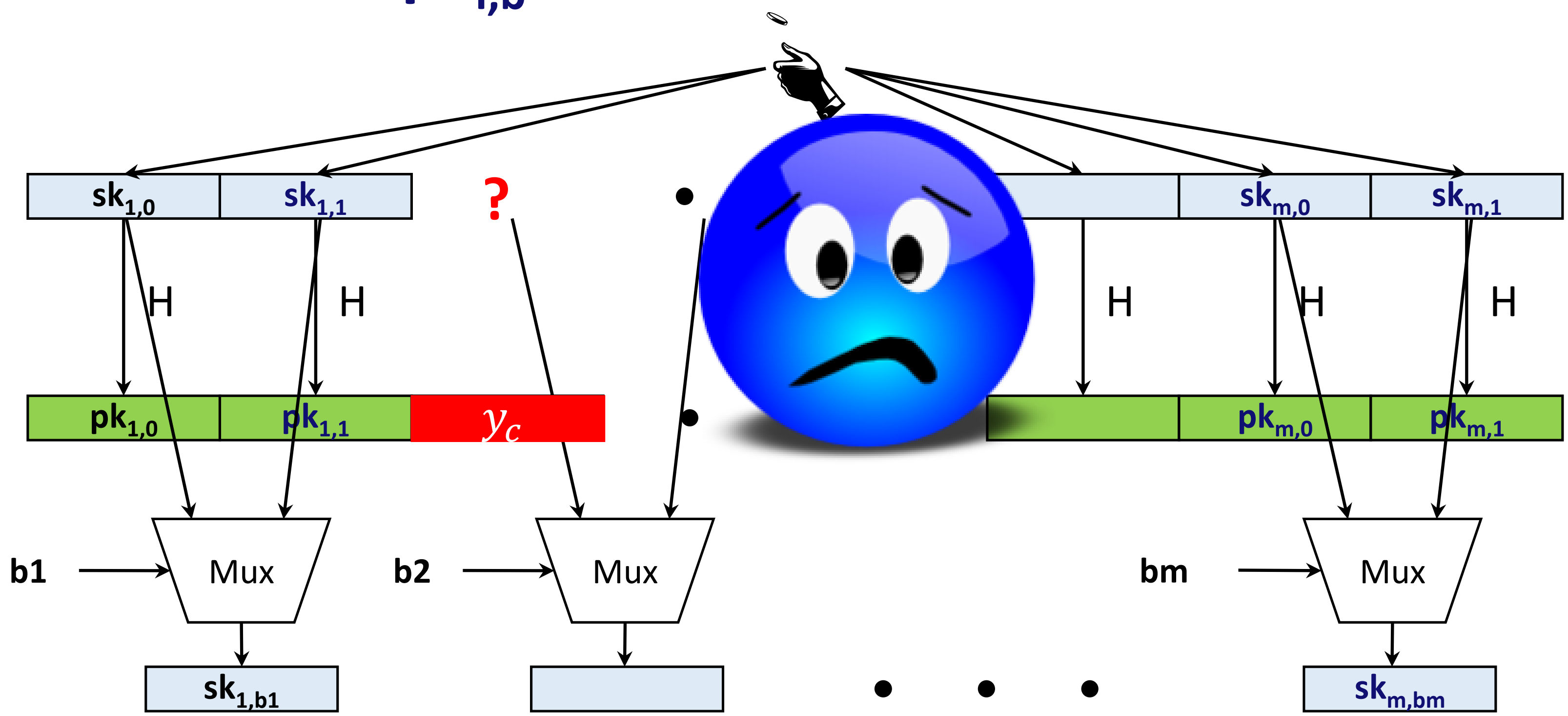
Set $H \leftarrow h_k$

Replace random $\mathbf{pk_{i,b}}$

Adv. Message: M = b1,…,bm

If bi = b return fail

else return Sign(M)



Reduction

Input: y_c, k

Set $H \leftarrow h_k$

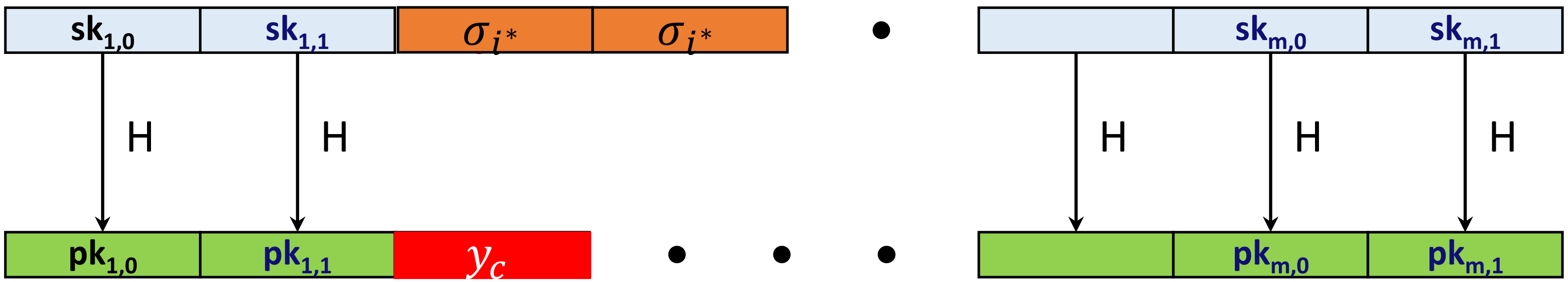
Choose random **$pk_{i,b}$**

Forgery: M* = b1*,…,bm*,

$\sigma = \sigma_1, \ldots, \sigma_m$

If bi $\neq$ b return fail

Else return σ_{i^*}



Reduction - Analysis

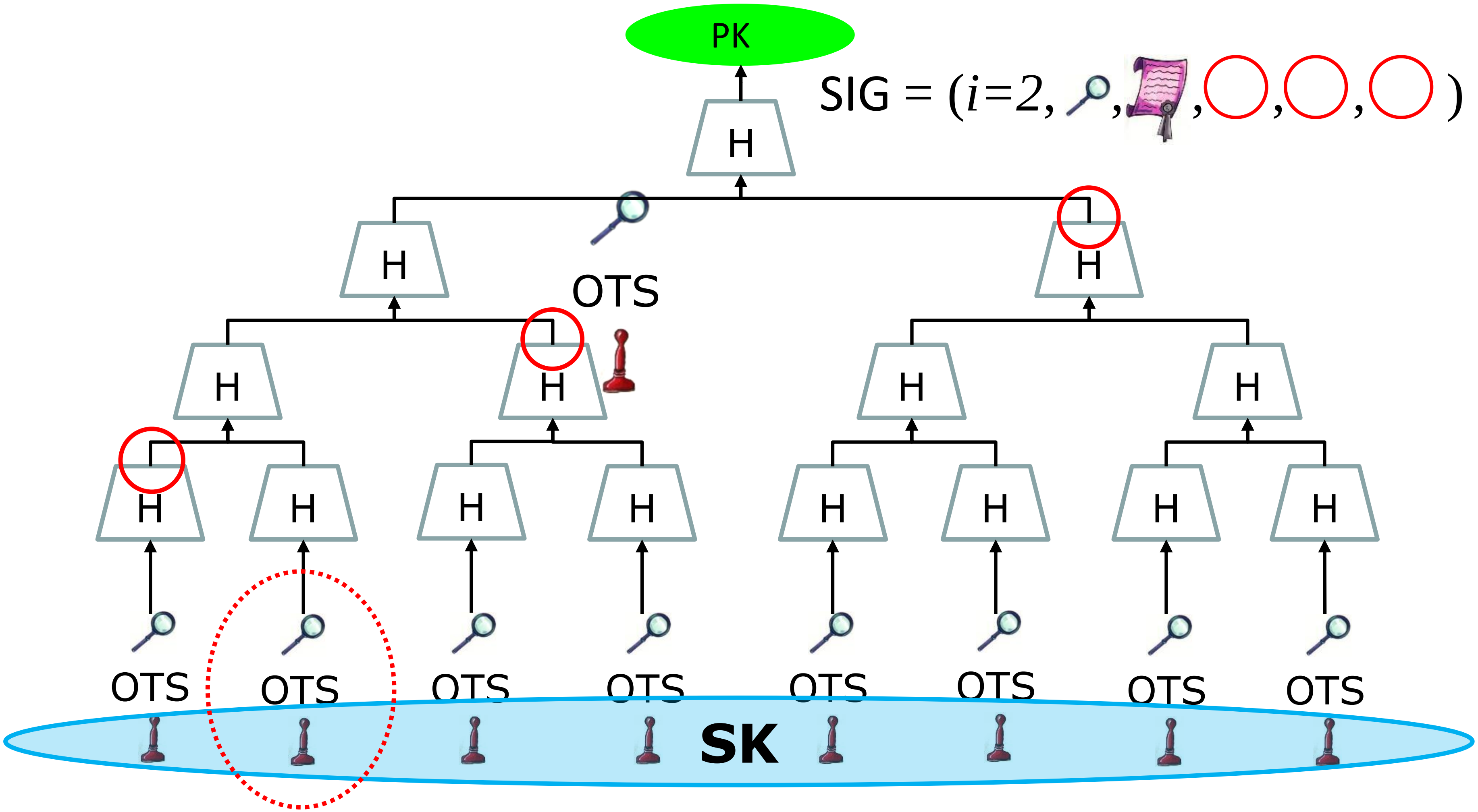
Abort in two cases:

1. bi = b
   probability ½ : b is a random bit
2. bi $\neq$ b

   probability 1 - 1/m: At least one bit has to flip as M* $\neq$ M

Reduction succeeds with A‘s success probability times 1/2m.

Merkle's Hash-based Signatures



Security

Theorem:

MSS is eu-cma-secure if OTS is a one-time eu-cma secure signature scheme and H is a random element from a family of collision resistant hash functions.

Reduction

Input: k, pk_{OTS}

1. Choose random $0 \leq i < 2^h$
2. Generate key pair using pk_{OTS} as ith OTS public key and $H \leftarrow h_k$
3. Answer all signature queries using sk or sign oracle (for index i)
4. Extract OTS-forgery or collision from forgery

Reduction (Step 4, Extraction)

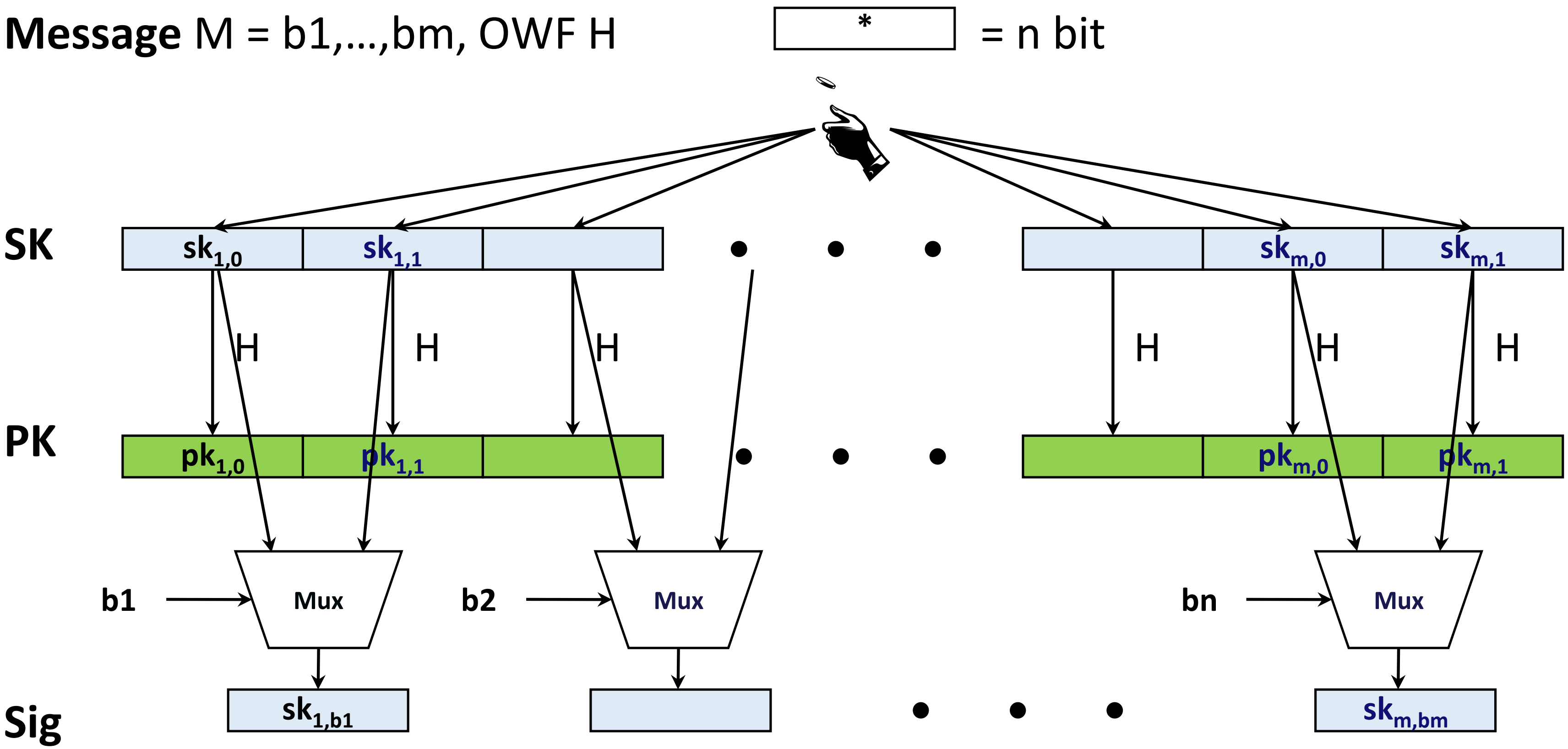
Forgery: ($i^*, \sigma_{OTS}^*, pk_{OTS}^*$, AUTH)

1. If pk_{OTS}^* equals OTS pk we used for i^* OTS, we got an OTS forgery.
   - Can only be used if $i^* = i$.
2. Else adversary used different OTS pk.
   - Hence, different leaves.
   - Still same root!
   - Pigeon-hole principle: Collision on path to root.

Winternitz-OTS

Recap LD-OTS [Lam79]

Message M = b1,...,bm, OWF H



LD-OTS in MSS

SIG = (i=2, , , , ,)

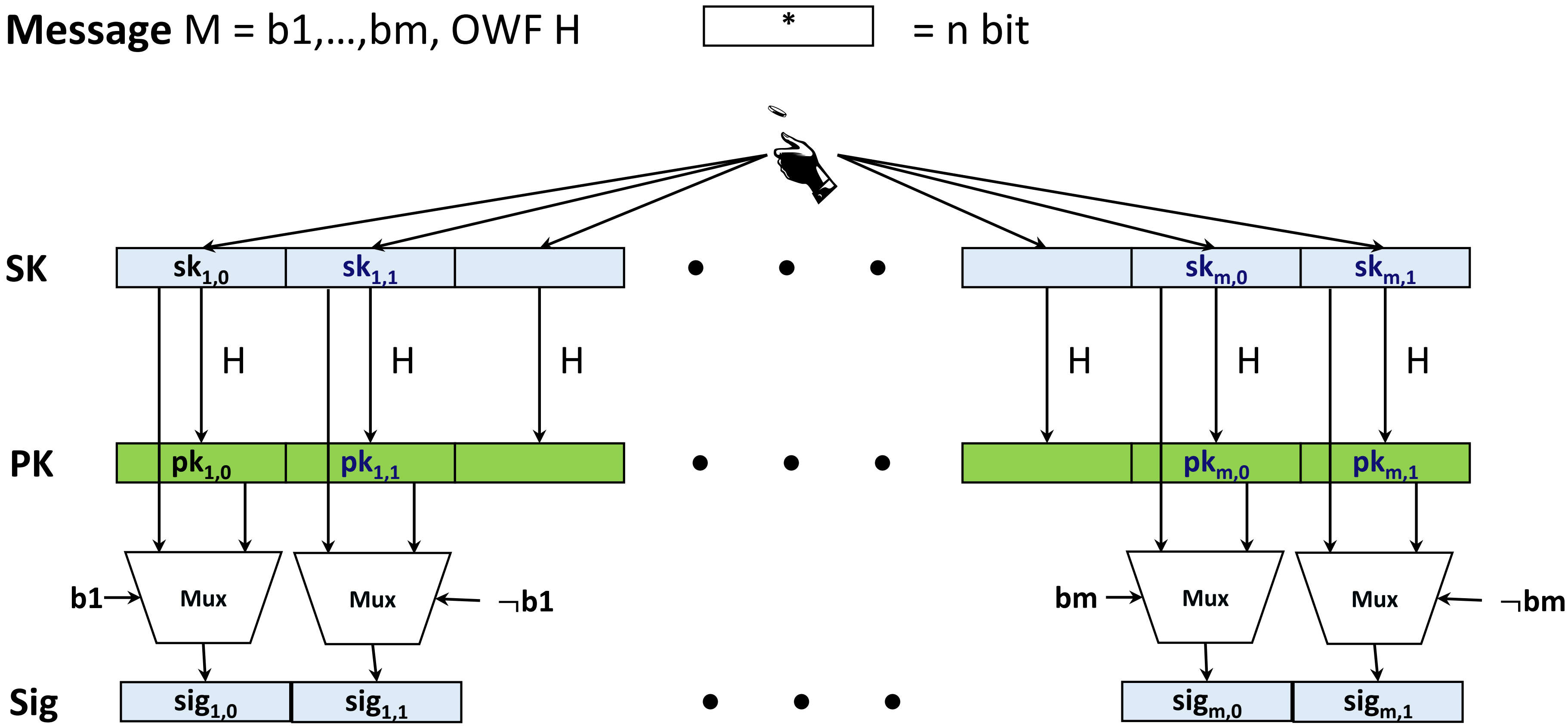
Verification:

1. Verify 
2. Verify authenticity of 

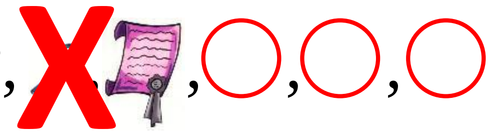
We can do better!

Trivial Optimization

Message M = b1,…,bm, OWF H



Optimized LD-OTS in MSS

SIG = (i=2, )

Verification:

1. Compute  from 
2. Verify authenticity of 

Steps 1 + 2 together verify 

Germans love their „Ordnung"!

Message $M = b_1,\dots,b_m$, OWF H

SK: $sk_1,\dots,sk_m,sk_{m+1},\dots,sk_{2m}$

PK: $H(sk_1),\dots,H(sk_m),H(sk_{m+1}),\dots,H(sk_{2m})$

Encode M: $M' = M || \neg M$ **=** $b_1,\dots,b_m,\neg b_1,\dots,\neg b_m$
(instead of $b_1, \neg b_1,\dots,b_m,\neg b_m$)

Sig: sig_i **=** $\begin{cases} sk_i & \text{, if } b_i = 1 \\ H(sk_i) & \text{, otherwise} \end{cases}$

Checksum with bad performance!

Optimized LD-OTS

Message M = $b_1,\ldots,b_m$, OWF H

SK: $sk_1,\ldots,sk_m,sk_{m+1},\ldots,sk_{m+\log m}$

PK: $H(sk_1),\ldots,H(sk_m),H(sk_{m+1}),\ldots,H(sk_{m+\log m})$

Encode M: M' = $b_1,\ldots,b_m,\neg \sum_1^m b_i$

Sig: sig_i **=** $\begin{cases} sk_i & \text{, if } b_i = 1 \\ H(sk_i) & \text{, otherwise} \end{cases}$

IF one b_i is flipped from 1 to 0, another b_j will flip from 0 to 1

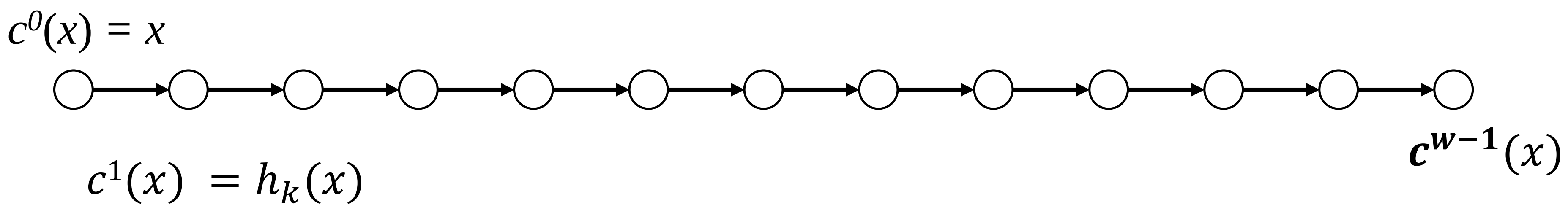
Function chains

Function family: $H_n := \{h_k : \{0,1\}^n \to \{0,1\}^n\}$

$h_k \xleftarrow{\$} H_n$

Parameter w

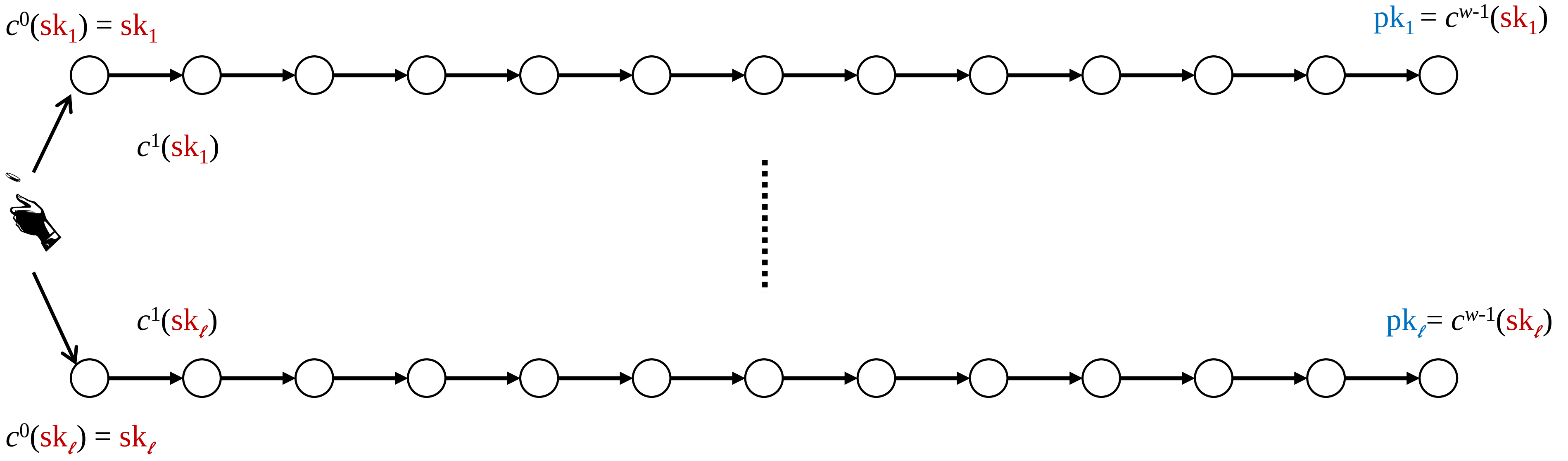
Chain: $c^i(x) = h_k(c^{i-1}(x)) = \underbrace{h_k \circ h_k \circ \ldots \circ h_k(x)}_{i-times}$



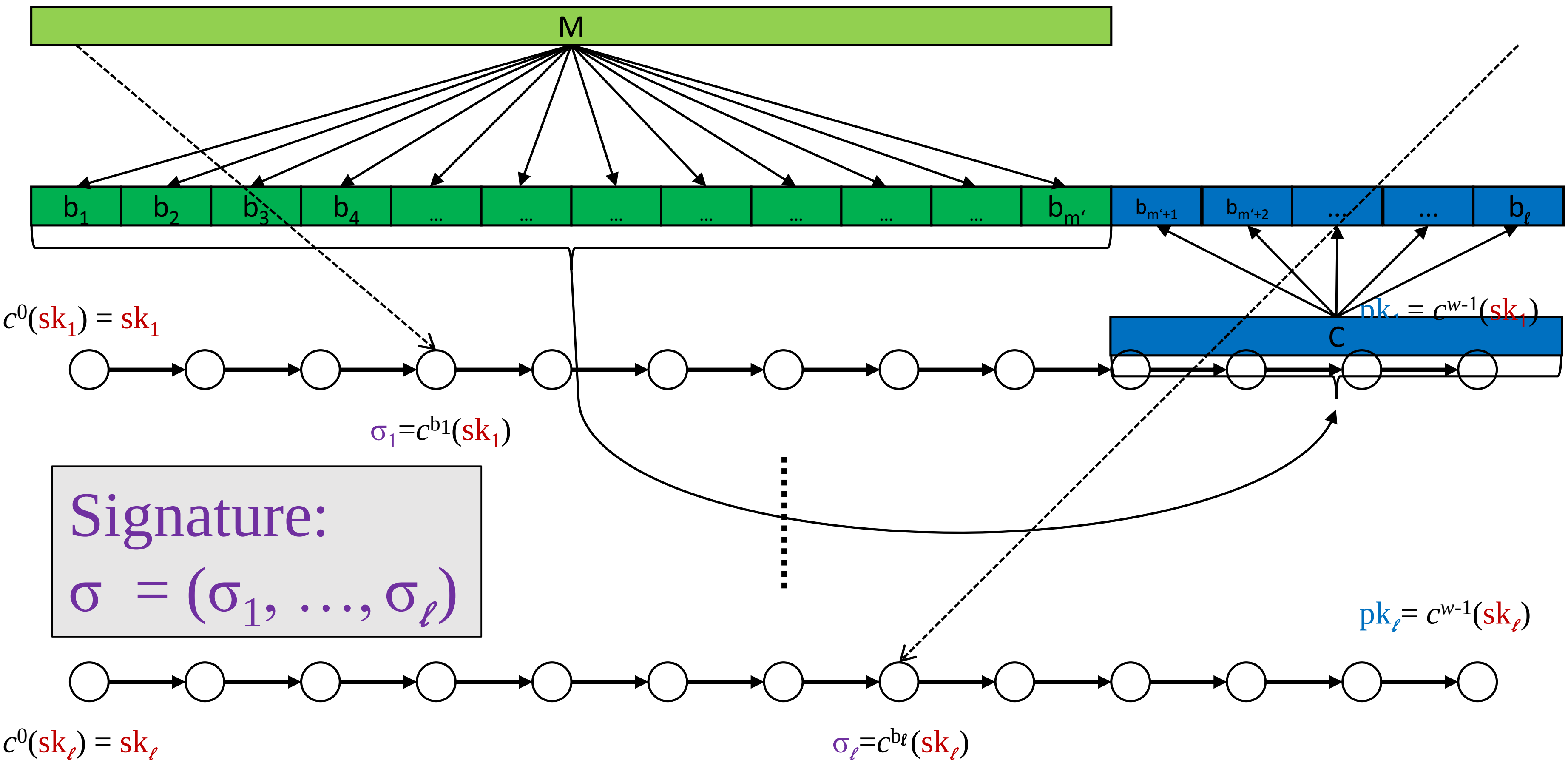
WOTS

Winternitz parameter $\boldsymbol{w}$, security parameter $\boldsymbol{n}$, message length $\boldsymbol{m}$, function family H_n

Key Generation: Compute l, sample h_k

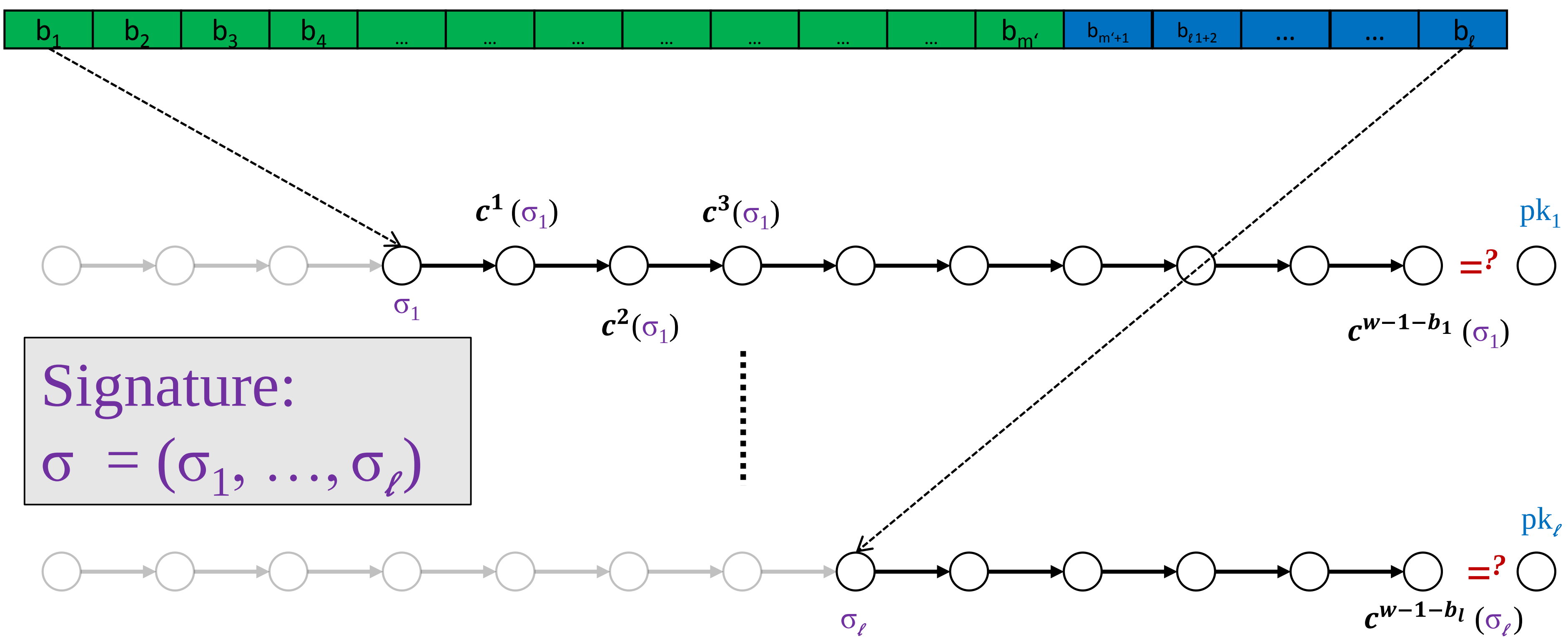


WOTS Signature generation



WOTS Signature Verification

Verifier knows: M, w



WOTS Function Chains

For $x \in \{0,1\}^n$ define $c^0(x) = x$ and

- WOTS: $c^i(x) = h_k(c^{i-1}(x))$
- WOTS$: $c^i(x) = h_{c^{i-1}(x)}(r)$
- WOTS+: $c^i(x) = h_k(c^{i-1}(x) \oplus r_i)$

WOTS Security

Theorem (informally):

W-OTS *is strongly unforgeable under chosen message attacks if* H_n *is a collision resistant family of undetectable one-way functions.*

W-OTS$^{\$}$ *is existentially unforgeable under chosen message attacks if* H_n *is a pseudorandom function family.*

W-OTS^{+} *is strongly unforgeable under chosen message attacks if* H_n *is a 2nd-preimage resistant family of undetectable one-way functions.*

XMSS

XMSS

Tree: Uses bitmasks

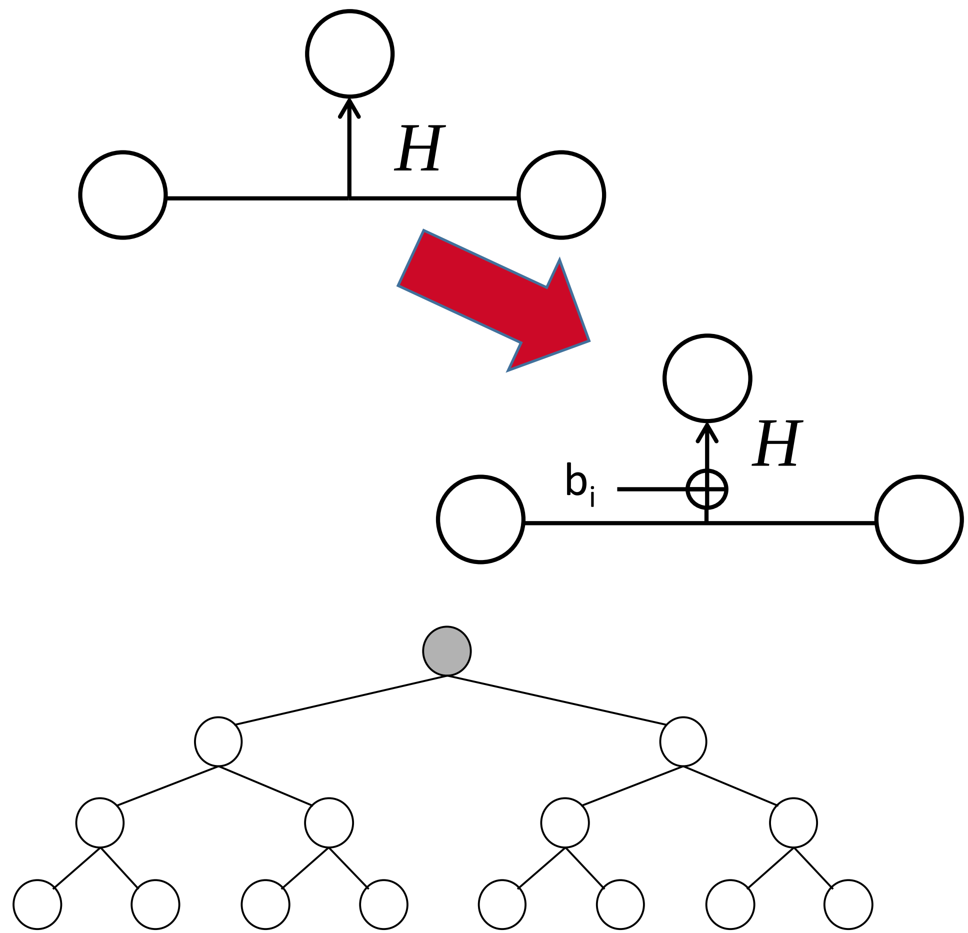
Leafs: Use binary tree with bitmasks

OTS: WOTS$^+$

Mesage digest: Randomized hashing

Collision-resilient

-> signature size halved

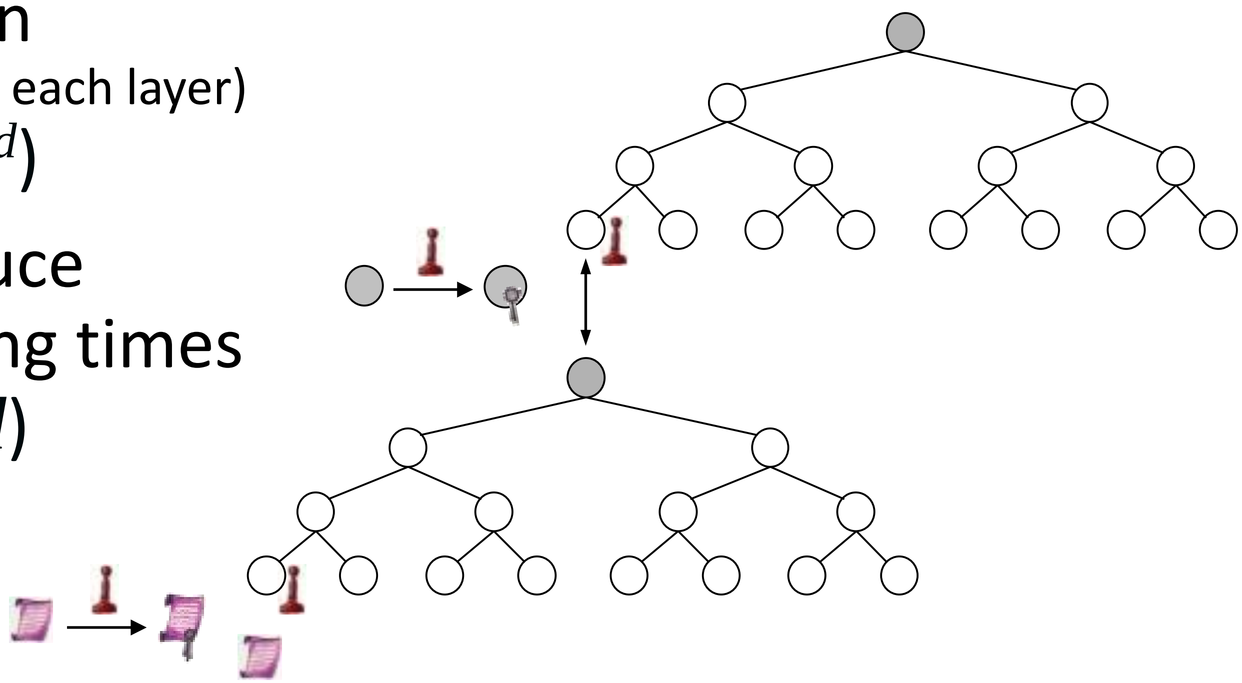


Multi-Tree XMSS

Uses multiple layers of trees

-> Key generation
(= Building first tree on each layer)
$\Theta(2^h) \longrightarrow \Theta(d*2^{h/d})$

-> Allows to reduce
worst-case signing times
$\Theta(h/2) \longrightarrow \Theta(h/2d)$



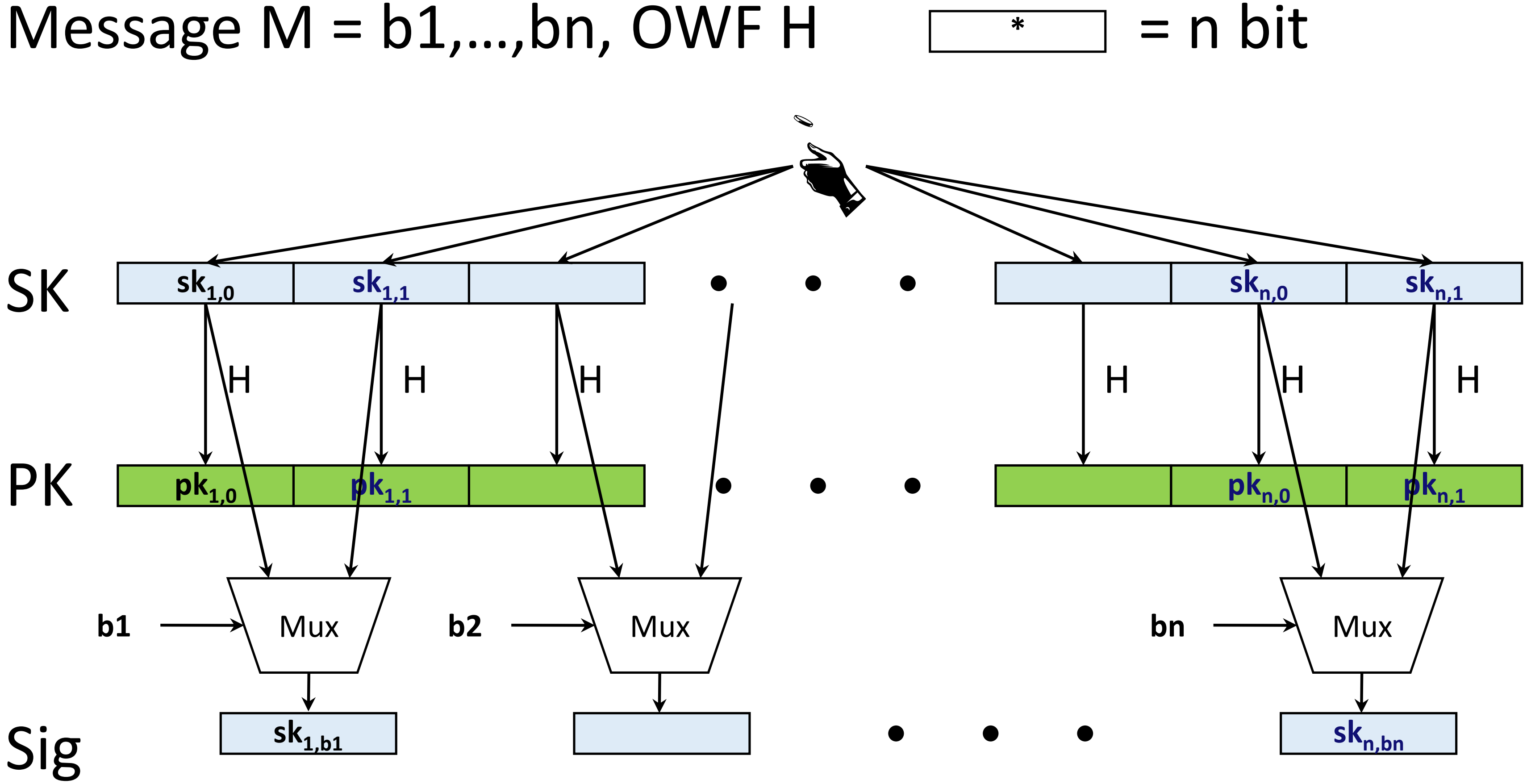
ELIMINATE THE STATE

Protest?

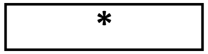
Few-Time Signature Schemes

Recap LD-OTS

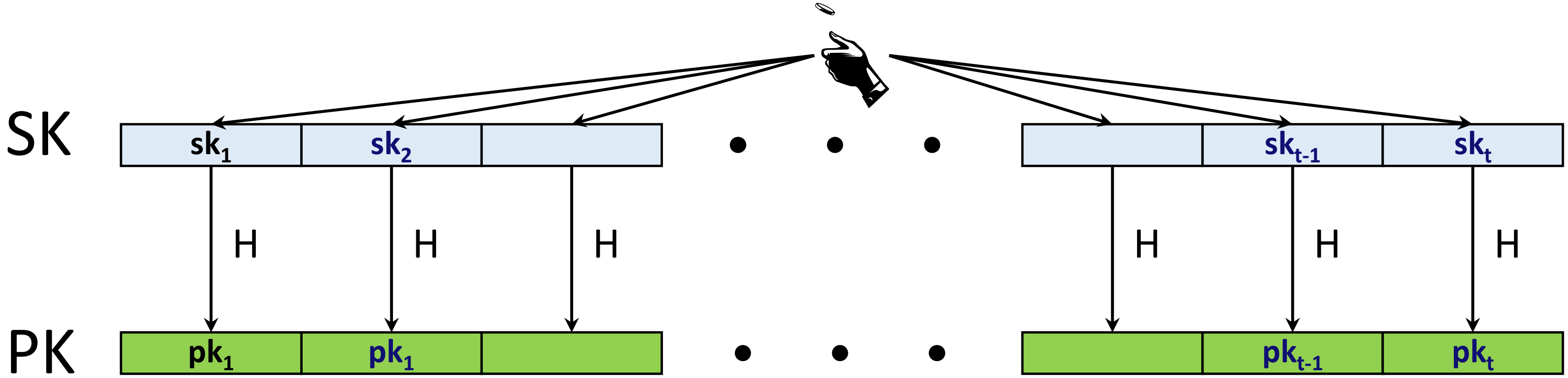
Message M = b1,...,bn, OWF H



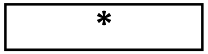
HORS [RR02]

Message M, OWF H, CRHF H’  = n bit

Parameters $t=2^a$,k, with m = ka (typical a=16, k=32)



HORS mapping function

Message M, OWF H, CRHF H’  = n bit

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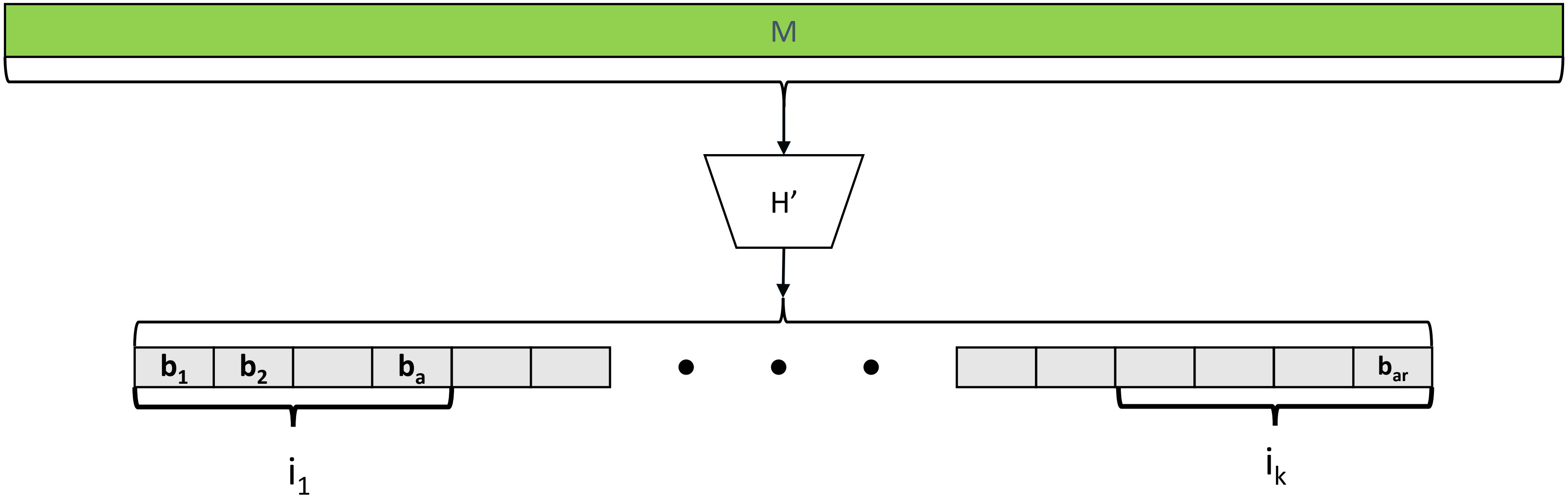
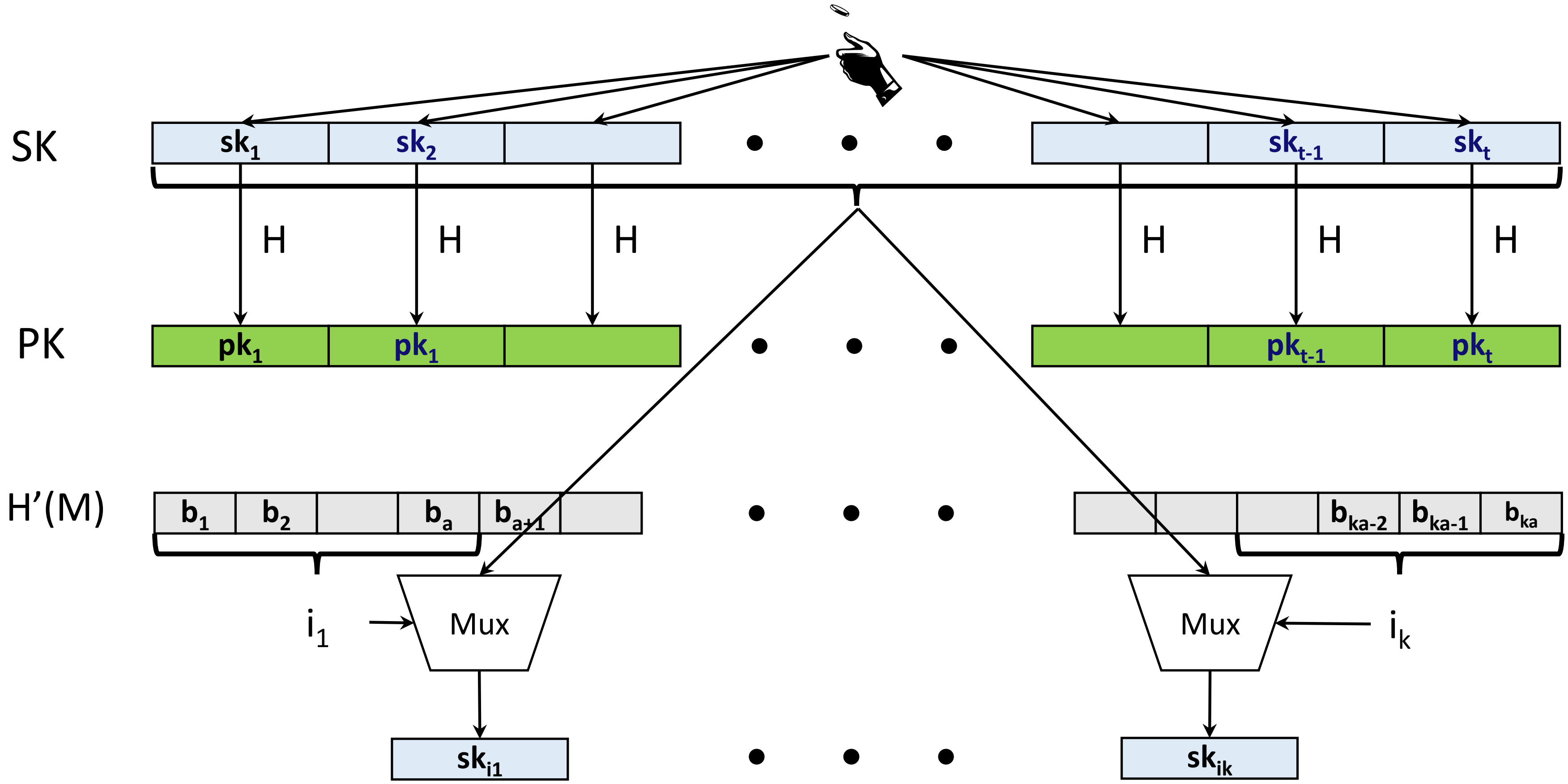


HORS

Message M, OWF H, CRHF H' [*] = n bit

Parameters $t=2^a$,k, with m = ka (typical a=16, k=32)



HORS Security

- M mapped to k element index set $M^i \in \{1, \dots, t\}^k$
- Each signature publishes k out of t secrets
- Either break one-wayness or…

- r-Subset-Resilience: After seeing index sets M_j^i for r messages $msg_j, 1 \le j \le r$, hard to find $msg_{r+1} \neq msg_j$ such that $M_{r+1}^i \in \bigcup_{1 \le j \le r} M_j^i$.

- Best generic attack: $\mathrm{Succ}_{\text{r-SSR}}(A, q) = q \left(\frac{rk}{t}\right)^k$

$\longrightarrow$ Security shrinks with each signature!

HORST

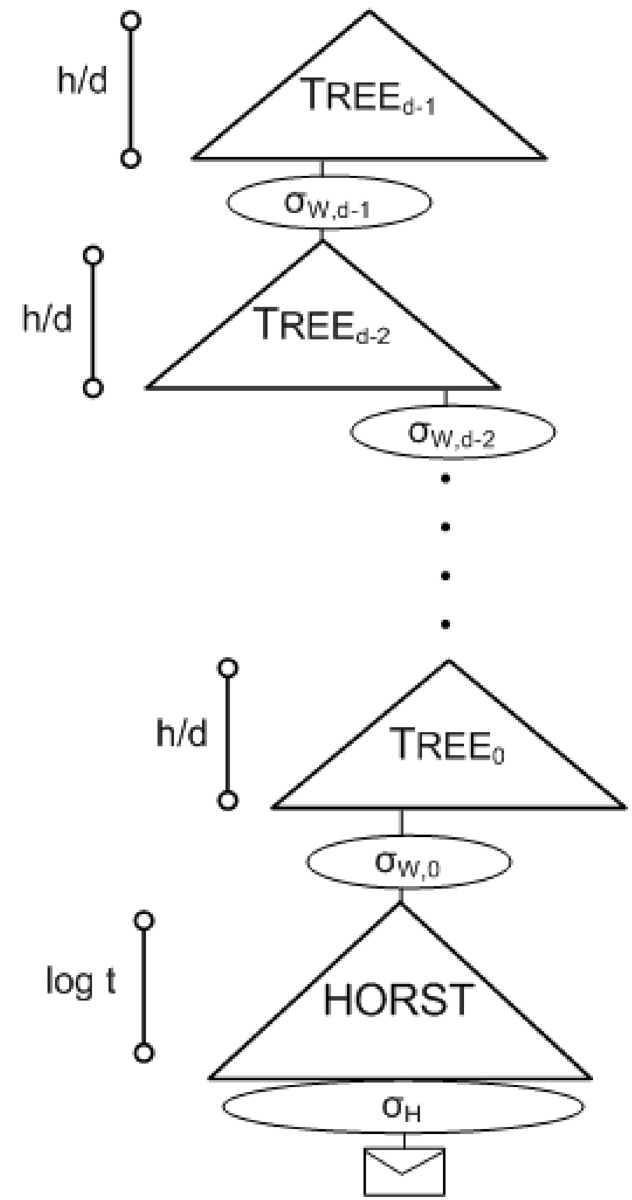
Using HORS with MSS requires adding PK (tn) to MSS signature.

HORST: Merkle Tree on top of HORS-PK

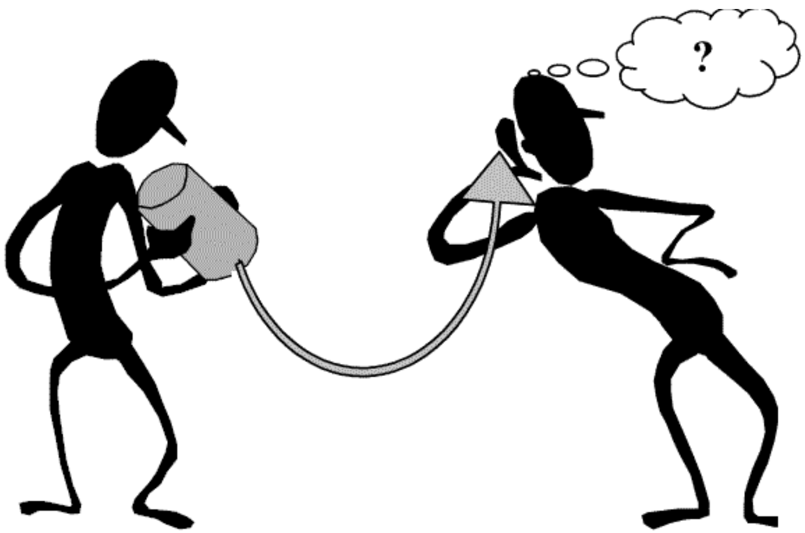
- New PK = Root
- Publish Authentication Paths for HORS signature values
- PK can be computed from Sig
- With optimizations: $tn \rightarrow (k(\log t - x + 1) + 2^x)n$
    - E.g. SPHINCS-256: 2 MB → 16 KB
- Use randomized message hash

SPHINCS

- Stateless Scheme
- XMSS^{MT} + HORST
  + (pseudo-)random index
- Collision-resilient
- Deterministic signing
- SPHINCS-256:
  - 128-bit post-quantum secure
  - Hundrest of signatures / sec
  - 41 kb signature
  - 1 kb keys



Thank you!
Questions?



For references & further literature see
https://huelsing.wordpress.com/hash-based-signature-schemes/literature/