Bad directions in cryptographic hash functions

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Challenge announced at CRYPTO 2014 rump session:

Page 11 from `http://crypto.2014.rump.cr.yp.to/bca480a4e7fcdaf5bfa9dec75ff890c8.pdf`:

Everybody loves (virtual black-box / indistinguishability) obfuscation... so we implemented it!

Implementation combines ideas from various obfuscation papers and uses CLT multilinear map scheme

It is slow... but not as slow as you might think

Example: To obfuscate a 16-bit point function (i.e., 16 OR gates) with 52 bits of security using an Amazon EC2 machine with 32 cores:

- Obfuscation time: $\approx$ 7 hours
- Evaluation time: $\approx$ 3 hours
- Obfuscation size: 31 GB

$\Longrightarrow$ **it's almost nearly practical**

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Page 14 from `http://crypto.2014.rump.cr.yp.to/bca480a4e7fcdaf5bfa9dec75ff890c8.pdf`:

Code is available: `https://github.com/amaloz/ind-obfuscation`

ePrint version should be up at some point

For the cryptanalysts in the audience: We have an obfuscated 14-bit point function on Dropbox[1] — learn the point and you win!

[1] `https://www.dropbox.com/s/85d03o0ny3b1c0c/point-14.circ.obf.60.zip`

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Breaking the challenge:

- 14-bit point function,
- "60 bits of security",
- obfuscation size: 25 GB.

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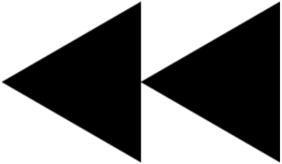
Further procrastination (“this is fast enough”)	about half a week
Our faster program evaluating all inputs on 22 PCs	34 minutes
Second successful break of challenge on 21 PCs	19 minutes

What happened?



Restart from the beginning

- (Eurocrypt 2013) Garg, Gentry, and Halevi, *Candidate multilinear maps from ideal lattices.* (GGH)
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- Bunch of applications of iO
  - Previously impossible constructions.
  - Reconstructing old things more inefficiently

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- Bunch of applications of iO
  - Previously impossible constructions.
  - Reconstructing old things more inefficiently *BUT WITH iO!*

The University of Maryland Implementation

- (eprint 2014) Apon, Huang, Katz, and Malozemoff, *Implementing cryptographic program obfuscation.*
- First implementation of candidate obfuscation
- Implement AGIS (CCS 2014) scheme with CLT
- Biggest circuit: 16 bit point function (16 OR gates)

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- First implementation of candidate obfuscation
- Implement AGIS (CCS 2014) scheme with CLT
- Biggest circuit: 16 bit point function (16 OR gates)
- Reason? Bad performance!
  - Obfuscation time $\approx$ 7 hours
  - Evaluation time $\approx$ 3 hours
  - Obfuscation size: 31 GB

Point-function obfuscation - Obfuscation

Obfuscation for n-bit point, security parameter λ:

- modulus $q \in \mathbb{N}$ (having $\Theta((\lambda n)^2 \log_2 \lambda)$ bits).
- $2n$ matrices $B_{b,k} \in \mathbb{Z}_q^{(n+2)\times(n+2)}$ for $1 \leq b \leq n$ and $k \in \{0,1\}$,
- $s, t \in \mathbb{Z}_q^{(n+2)}$,
- zero test value $p_{\mathrm{zt}} \in \mathbb{Z}_q$ (Multi-linear map artifact).

Point-function obfuscation - Evaluation

Evaluation for $x = (x[1], x[2], \ldots, x[n]) \in \{0, 1\}^n$

- Compute $A = B_{1,x[1]} B_{2,x[2]} \cdots B_{n,x[n]}$.
- Compute $y(x) = s^\top A t$.
- Compute $y(x) p_{\mathrm{zt}}$ and reduce mod q to the range $[-(q-1)/2, (q-1)/2]$.
- Multiply the remainder by $2^{2\lambda+11}$, divide by q, and round to the nearest integer.
- Output 0 if result is 0; output 1 otherwise.

Performance & Security

Performance:

- Experimentally ($n = 14$): $\approx 2^{49}$ cycles / evaluation
- Theoretically:
  - $(n-1)(n+2)^3$ multiplications of integers $> (q-1)^n$ (on average) to compute A.
  - Total complexity $n^{7+o(1)}$ (assuming schoolbook multiplication).

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Security:

- Parameters chosen for 60 bit security of multi-linear map.
- "Exhaustive search too slow" ($> 2^{60}$ cycles for $n = 14$).

Our attack

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1. Speed up evaluation
   1.1 Intermediate reductions mod q
   1.2 Only matrix-vector products
2. Speed up exhaustive search
   2.1 Reuse intermediate results
   2.2 Meet-in-the-middle

Intermediate reductions mod q

- Integers grow up to $(n+2)^{n-1}(q-1)^n$; typically larger than $(q-1)^n$.
- Multiplication essentially linear in $\#$bits ($b^{1+o(1)}$).
- $y(x)p_{\mathrm{zt}}$ is in $\mathbb{Z}_q$.

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Improvement:

- Reduce mod q after every vector-vector product.
- Inputs to mult $< q-1$.
- Reduce costs by factor n:

$$n^{7+o(1)} \Rightarrow n^{6+o(1)}$$

Only matrix-vector products

Step 1 of eval: Compute $A = B_{1,x[1]} B_{2,x[2]} \cdots B_{n,x[n]}$.
Step 2 of eval: Compute $y(x) = s^\top A t$.

Only matrix-vector products

Step 1 of eval: Compute $A = B_{1,x[1]} B_{2,x[2]} \cdots B_{n,x[n]}$.
Step 2 of eval: Compute $y(x) = s^\top A t$.

- Matrix-matrix products are unnecessary!
- $y(x) = \left(\cdots\left(\left(s^\top B_{1,x[1]}\right) B_{2,x[2]}\right) \cdots B_{n,x[n]}\right) t$
- $(n-1)(n+2)^3$ multiplications $\Rightarrow (n-1)(n+2)^2$ (omitting vector-vector)

$$n^{6+o(1)} \Rightarrow n^{5+o(1)}$$

From single evaluation to exhaustive search

So far we reduced time for one evaluation

$$n^{7+o(1)} \Rightarrow n^{5+o(1)},$$

dominated by $(n+1)^2$ dot products mod q.
This means, exhaustive search takes

$$(n+1)^2 2^n$$

dot products mod q.

Reuse intermediate results

- $y(\mathbf{0}) = \left(\cdots\left((s^\top B_{1,0})B_{2,0}\right)\cdots B_{n-1,0}\right)\cdots B_{n,\color{red}{0}}\, t$
- $y(\mathbf{1}) = \left(\cdots\left((s^\top B_{1,0})B_{2,0}\right)\cdots B_{n-1,0}\right)\cdots B_{n,\color{red}{1}}\, t$

Reuse intermediate results

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- Generally, trying all inputs in order, average cost to update result: 2 vector-matrix products.

Improvement:

$$(n+1)^2 2^n \Rightarrow 2(n+1)2^n \text{ dot products mod } q$$

Meet-in-the-middle

- Split computation into two halves

$$y(x) = (s^\top B_{1,x[1]} \cdots B_{\ell,x[\ell]})(B_{\ell+1,x[\ell+1]} \cdots B_{n,x[n]} t).$$

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- precompute a table of "left" products

$$L[x[1], \ldots, x[\ell]] = sB_{1,x[1]} \cdots B_{\ell,x[\ell]}$$

for all 2^ℓ choices of $(x[1], \ldots, x[\ell])$

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- for each choice of $(x[\ell+1], \ldots, x[n])$, compute "right" product

$$R[x[\ell+1], \ldots, x[n]] = B_{\ell+1,x[\ell+1]} \cdots B_{n,x[n]} t,$$

  and multiply each element of the L table by this vector.

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Improvement for $\ell = n/2$ (assuming n is even):

$$2(n+1)2^n \Rightarrow 4(n+2)(2^{n/2}-1) + 2^n \text{ dot products mod } q$$

Summing up

We reduced time for exhaustive search from

$$n^{7+o(1)}2^n \Rightarrow n^{3+o(1)}2^n.$$

On one PC this takes 444.2 minutes. (Original program, estimated $= 7.6$ years)
On 22 PCs: 29.5 minutes.

Conclusion

- First two improvements speed up obfuscation scheme (though trivial).
- Reuse and meet-in-the-middle vulnerabilities inherent to obfuscation using matrix-branching (always at least factor n^2 speed-up).
- See paper for generalizations and further asymptotic speedups

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Thank you.