Cryptographic Hash Functions
Part II

Cryptography 1

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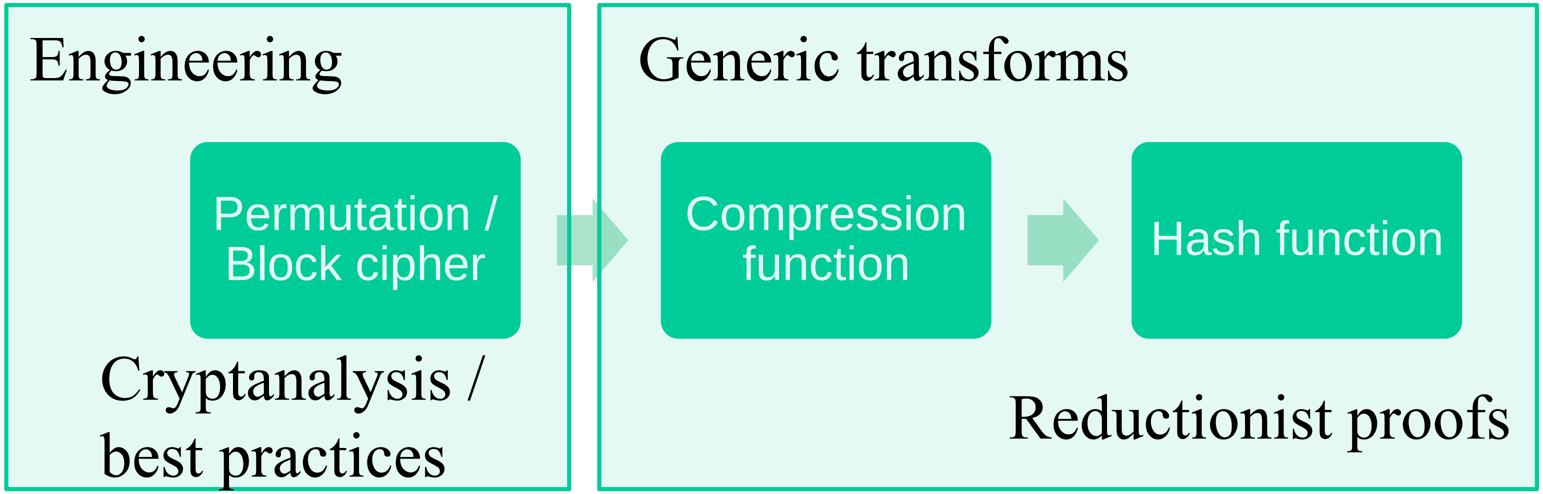
Hash function design

- Create fixed input size building block
- Use building block to build compression function
- Use „mode“ for length extension



Engineering

Permutation / Block cipher

Cryptanalysis / best practices

Generic transforms

Compression function

Hash function

Reductionist proofs

(LENGTH-EXTENSION) MODES

Merkle-Damgård construction

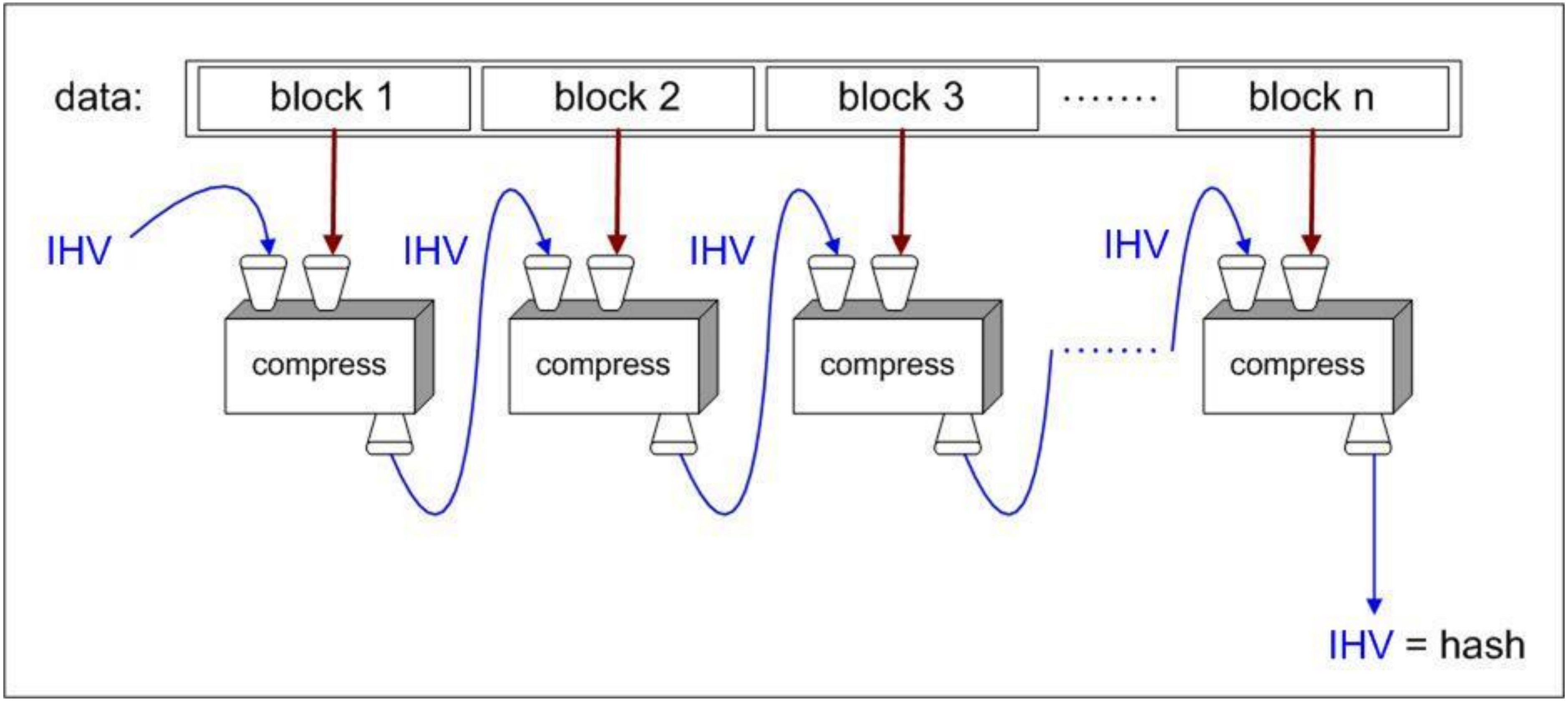
Given:

- ***compression function***: $CF : \{0,1\}^n \times \{0,1\}^r \rightarrow \{0,1\}^n$

Goal:

- ***Hash function:*** $H : \{0,1\}^* \rightarrow \{0,1\}^n$

Merkle-Damgård - iterated compression



Merkle-Damgård construction

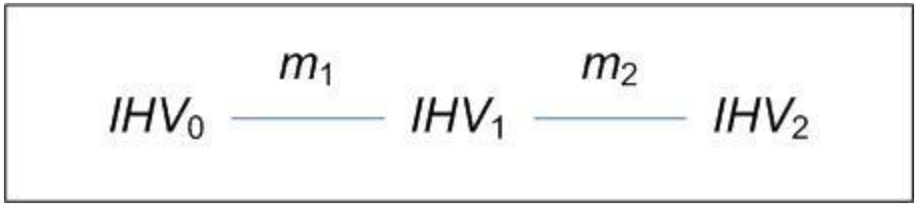
- assume that message m can be split up into blocks $m_1, \ldots, m_s$ of equal block length r
  - most popular block length is $r = 512$
- *compression function*: $CF : \{0,1\}^n \times \{0,1\}^r \rightarrow \{0,1\}^n$
- *intermediate hash values* (length n) as CF input *and* output
- *message blocks* as second input of CF
- start with fixed initial IHV_0 (a.k.a. IV = *initialization vector*)
- iterate CF : $IHV_1 = CF(IHV_0,m_1)$, $IHV_2 = CF(IHV_1,m_2)$, …, $IHV_s = CF(IHV_{s-1},m_s)$,
- take $h(m) = IHV_s$ as hash value
- advantages:
  - this design makes *streaming* possible
  - hash function analysis becomes compression function analysis
  - analysis easier because domain of CF is finite

padding

- ***padding***: add dummy bits to satisfy block length requirement
- **non-ambiguous padding: add one 1-bit and as many 0-bits as necessary to fill the final block**
  - **when original message length is a multiple of the block length, apply padding anyway, adding an extra dummy block**
  - **any other non-ambiguous padding will work as well**

Merkle-Damgård strengthening

- **let padding leave final 64 bits open**
- **encode in those 64 bits the original message length**
  - that's why messages of length $\geq 2^{64}$ are not supported
- **reasons:**
  - needed in the proof of the Merkle-Damgård theorem
  - prevents some attacks such as
    - trivial collisions for random *IV*

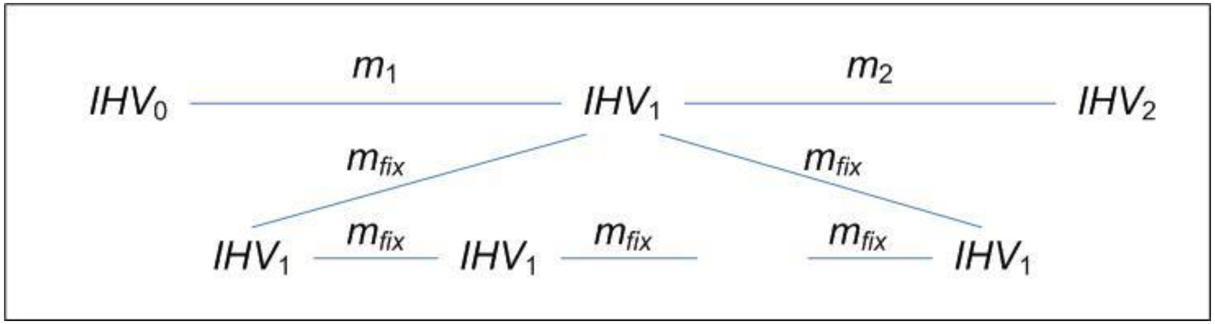


      - now $h(IHV_0,m_1||m_2) = h(IHV_1,m_2)$
    - see next slide for more

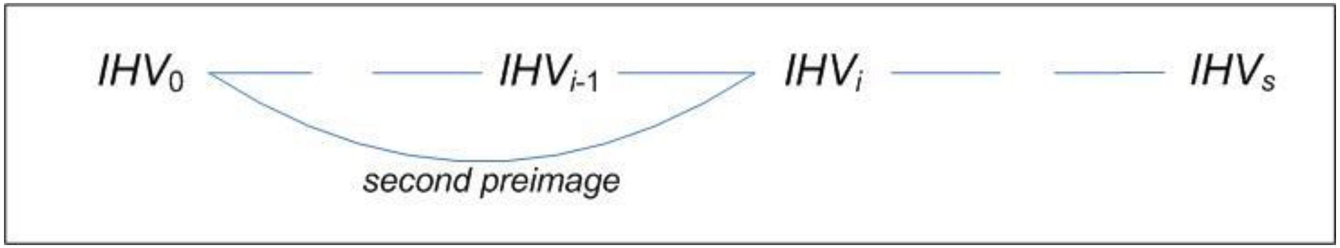
Merkle-Damgård strengthening, cont’d

- **fixpoint attack**

  fixpoint: *IHV*, *m* such that *CF*(*IHV*,*m*) = *IHV*



- **long message attack**



compression function collisions

- *collision* for a compression function: m_1, m_2, *IHV* such that $CF(IHV,m_1) = CF(IHV,m_2)$
- *pseudo-collision* for a compression function: m_1, m_2, IHV_1, IHV_2 such that $CF(IHV_1,m_1) = CF(IHV_2,m_2)$
- **Theorem** (Merkle-Damgård): If the compression function *CF* is pseudo-collision resistant, then a hash function *h* derived by Merkle-Damgård iterated compression is collision resistant.
  - Proof: Suppose $h(m_1) = h(m_2)$, then
    - If m_1, m_2 same size: locate the iteration where pseudo-collision occurs
    - Else a pseudo-collision for CF appears in the last blocks (cont. length)
- Note:
  - a method to find pseudo-collisions does not lead to a method to find collisions for the hash function
  - a method to find collisions for the compression function is almost a method to find collisions for the hash function, we ‘only’ have a wrong *IHV*

Sponges

Given:

- ***permutation***: $f : \{0,1\}^b \rightarrow \{0,1\}^b$

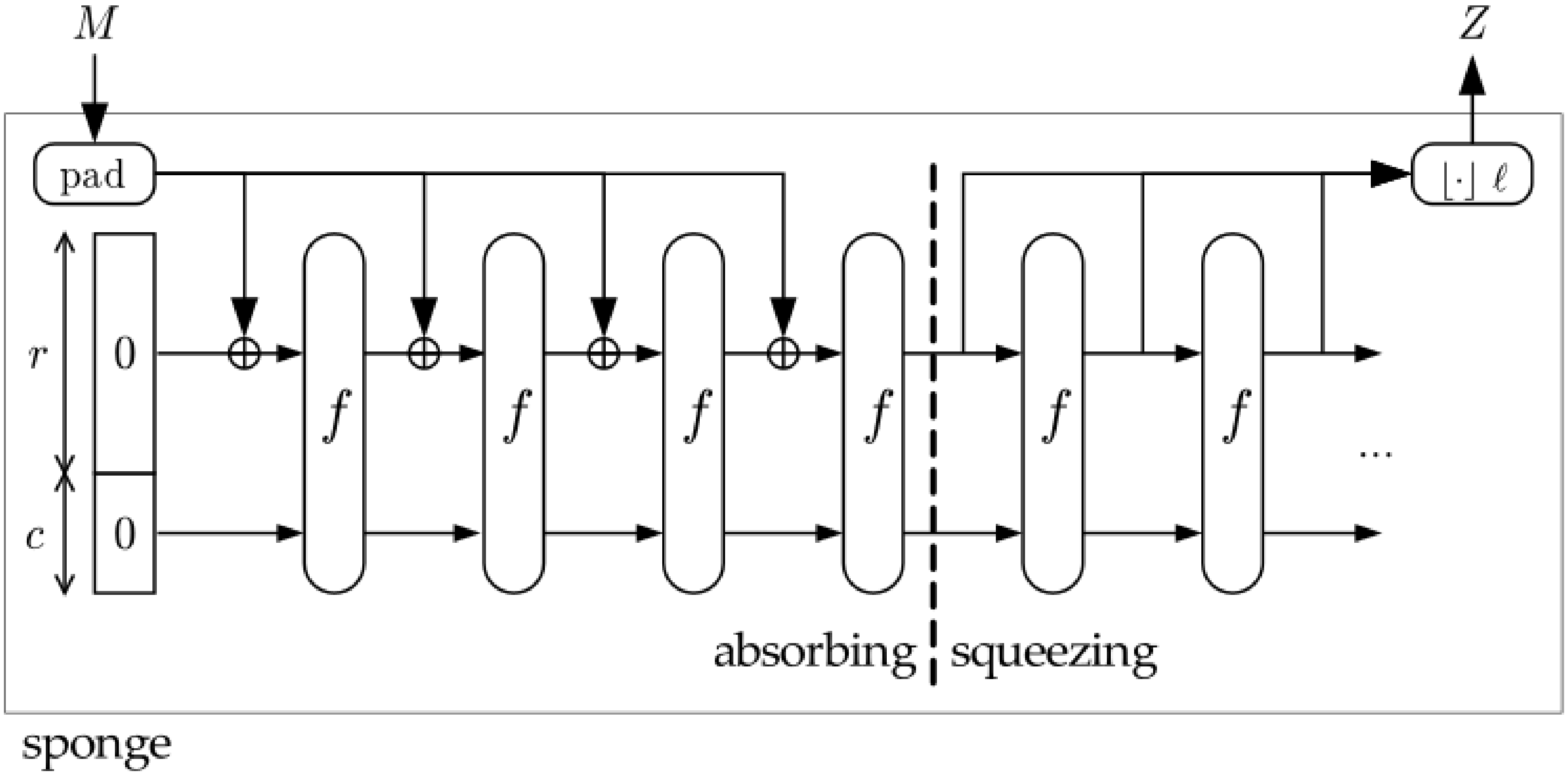
Goal:

- ***Hash function:*** $H : \{0,1\}^* \rightarrow \{0,1\}^n$
  (actually $H : \{0,1\}^* \rightarrow \{0,1\}^*$ **)**

- **(Already includes CF design, more later)**

Sponges

- **Used and introduced in SHA3 aka Keccak**
  - Guido Bertoni, Joan Daemen, Michaël Peeters and Gilles Van Assche



Intercourse: Random oracles

- Models the perfect hash function
- Truely random function without any structure
- Best attacks: Generic attacks (No structure available!)

Issue:

- No way to build a RO with polynomial description

Mind Model:

- Lazy-sampling
    - Imagine a black box implementing the function
    - For every new query, a random response is sampled
    - For old queries, former response is used

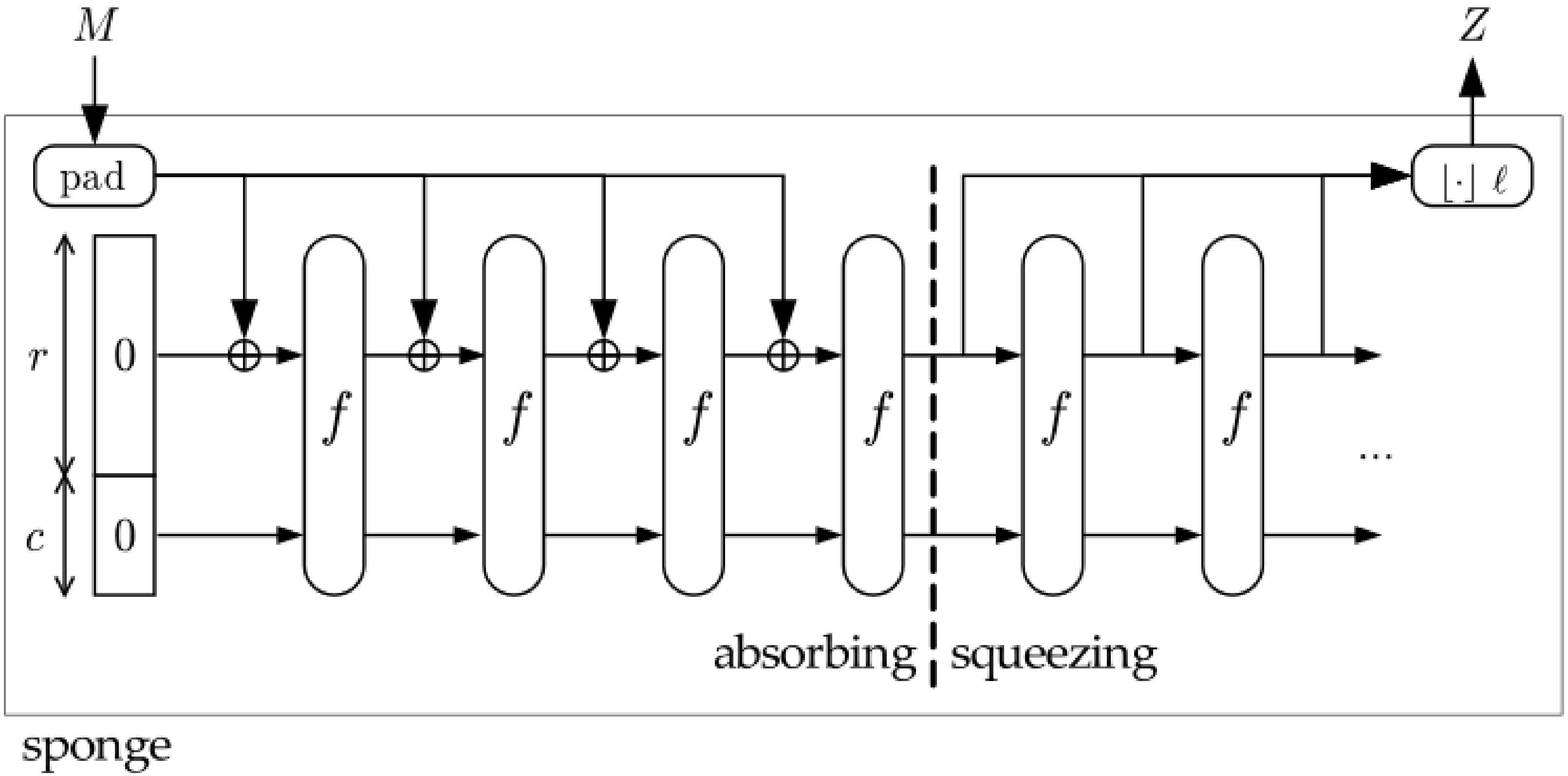
Sponge security

- **Theorem** **(Indifferentiability from a random oracle):**
  If f is a random permutation, the expected complexity for differentiating a sponge from a random oracle is $\sqrt{\pi}\,2^{c/2}$.

- **Note:**
  - Neat way to simplify security arguments
  - Implies bounds for all attacks that use less than $\sqrt{\pi}\,2^{c/2}$ queries
  - Bounds are those of generic attacks against a random oracle

Sponges

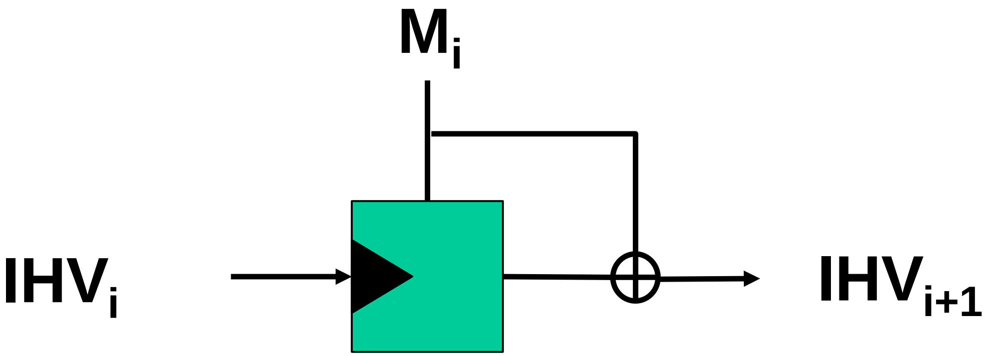
- **Used and introduced in SHA3 aka Keccak**
  - Guido Bertoni, Joan Daemen, Michaël Peeters and Gilles Van Assche



COMPRESSION FUNCTION DESIGN

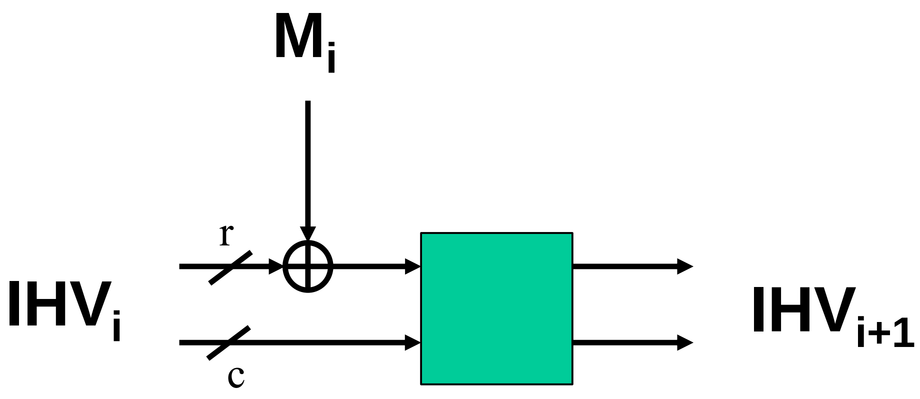
Block-Cipher-based designs

- Traditional approach
- Many possible modes
  - – see Preneel, Govaerts, Vandewalle. Hash functions based on block ciphers: a synthetic approach. CRYPTO’93
  - – security: Black, Rogaway, Shrimpton. Black-Box Analysis of the Block-Cipher-Based Hash-Function Constructions from PGV. CRYPTO’02
- Most popular: Matyas-Meyer-Oseas



Permutation-based designs

- **Less frequent use**
- **Keccak compression function:**



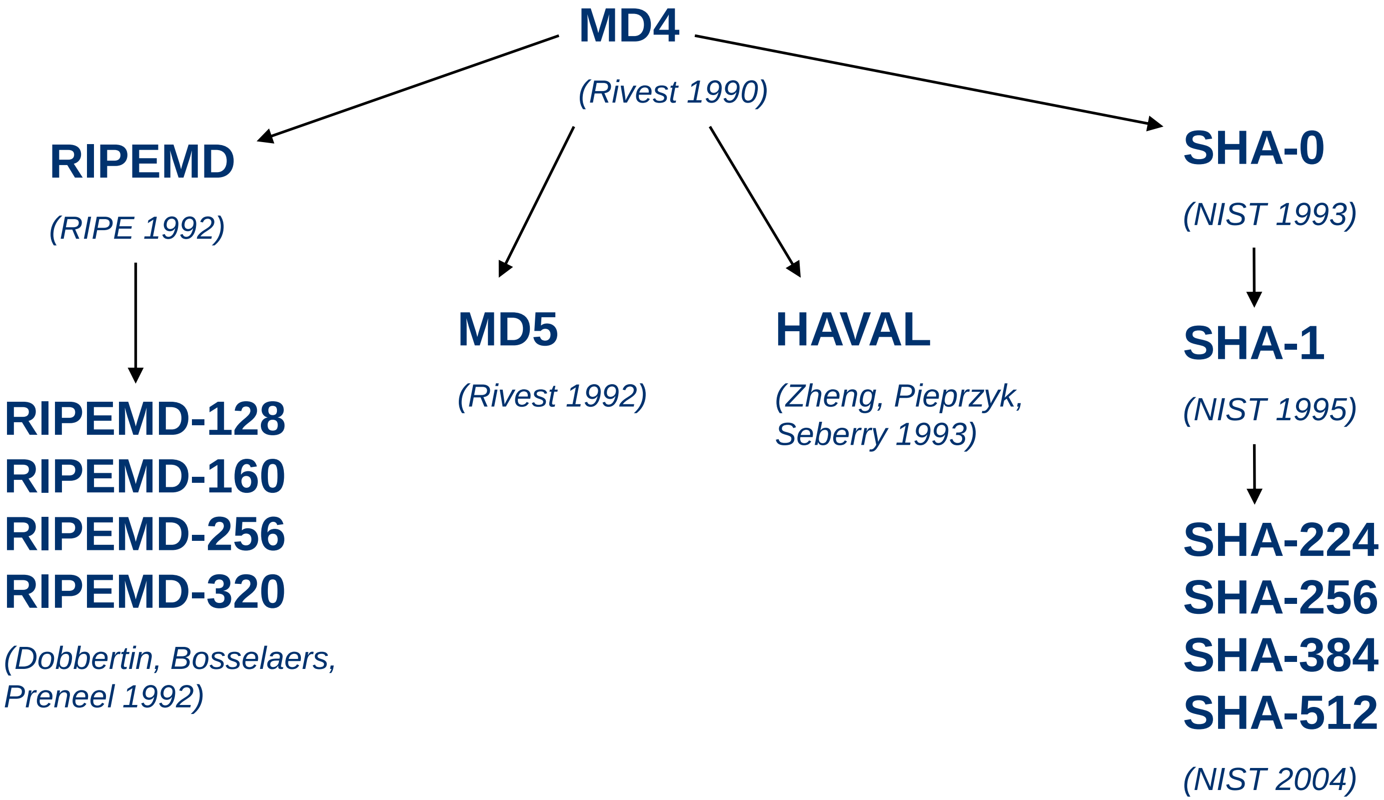
- **Important: NEVER hand out last c bits of IHV!**

Security

- **Generally analyzed in idealized models:**
  - „Black-box models“
  - Ideal cipher model
  - Random oracle model
  - Random permutation model
- **Proofs assuming underlying building block behaves like such an idealized building block**

BASIC BUILDING BLOCKS

the MD4 family of hash functions



MD4

(Rivest 1990)

RIPEMD

(RIPE 1992)

RIPEMD-128
RIPEMD-160
RIPEMD-256
RIPEMD-320

(Dobbertin, Bosselaers, Preneel 1992)

MD5

(Rivest 1992)

HAVAL

(Zheng, Pieprzyk, Seberry 1993)

SHA-0

(NIST 1993)

SHA-1

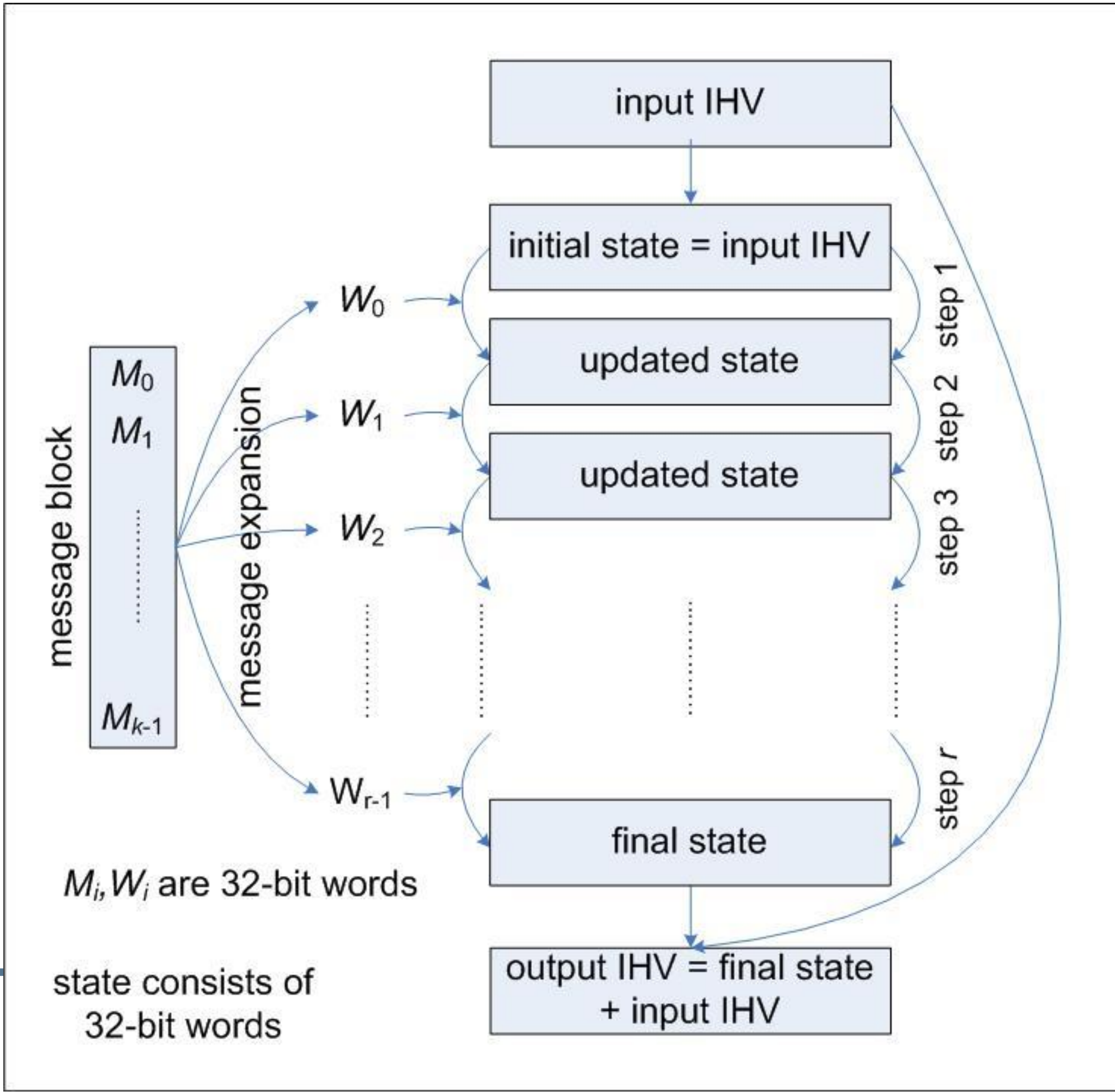
(NIST 1995)

SHA-224
SHA-256
SHA-384
SHA-512

(NIST 2004)

design of MD4 family compression functions

- **message block**
  - **split into words**
- **message expansion**
  - **input words for each step**
- ***IHV* → initial state**
- **each step updates state with an input word**
- **final state ‘added’ to *IHV***
  - **(feed-forward)**



design details

- **MD4, MD5, SHA-0, SHA-1 details:**
  - 512-bit message block split into 16 32-bit words
  - state consists of 4 (MD4, MD5) or 5 (SHA-0, SHA-1) 32-bit words
  - MD4: 3 rounds of 16 steps each, so 48 steps, 48 input words
  - MD5: 4 rounds of 16 steps each, so 64 steps, 64 input words
  - SHA-0, SHA-1: 4 rounds of 20 steps each, so 80 steps, 80 input words
  - message expansion and step operations use only very easy to implement operations:
    - bitwise Boolean operations
    - bit shifts and bit rotations
    - addition modulo 2^{32}
  - proper mixing believed to be cryptographically strong

message expansion

- MD4, MD5 use *roundwise permutation*, for MD5:
  - $W_0 = M_0, W_1 = M_1, \ldots, W_{15} = M_{15}$,
  - $W_{16} = M_1, W_{17} = M_6, \ldots, W_{31} = M_{12}$, (jump 5 mod 16)
  - $W_{32} = M_5, W_{33} = M_8, \ldots, W_{47} = M_2$, (jump 3 mod 16)
  - $W_{48} = M_0, W_{49} = M_7, \ldots, W_{63} = M_9$ (jump 7 mod 16)
- SHA-0, SHA-1 use *recursivity*
  - $W_0 = M_0, W_1 = M_1, \ldots, W_{15} = M_{15}$,
  - SHA-0: $W_i = W_{i-3}$ XOR W_{i-8} XOR W_{i-14} XOR W_{i-16} for i = 16, …, 79
  - problem: k^{th} bit influenced only by k^{th} bits of preceding words, so not much diffusion
  - SHA-1: $W_i = (W_{i-3}$ XOR W_{i-8} XOR W_{i-14} XOR $W_{i-16}) \lll 1$

    (additional rotation by 1 bit,

    this is the *only* difference between SHA-0 and SHA-1)

Example: step operations in MD5

- in each step only one state word is updated
- the other state words are *rotated* by 1
- state update:

  $A' = B + ((A + f_i(B,C,D) + W_i + K_i) \lll s_i)$

  K_i, s_i step dependent constants,

  $+$ is addition mod 2^{32},

  f_i round dependend boolean functions:

  $f_i(x,y,z) = xy \text{ OR } (\neg x)z$ for $i = 1, \ldots, 16$,

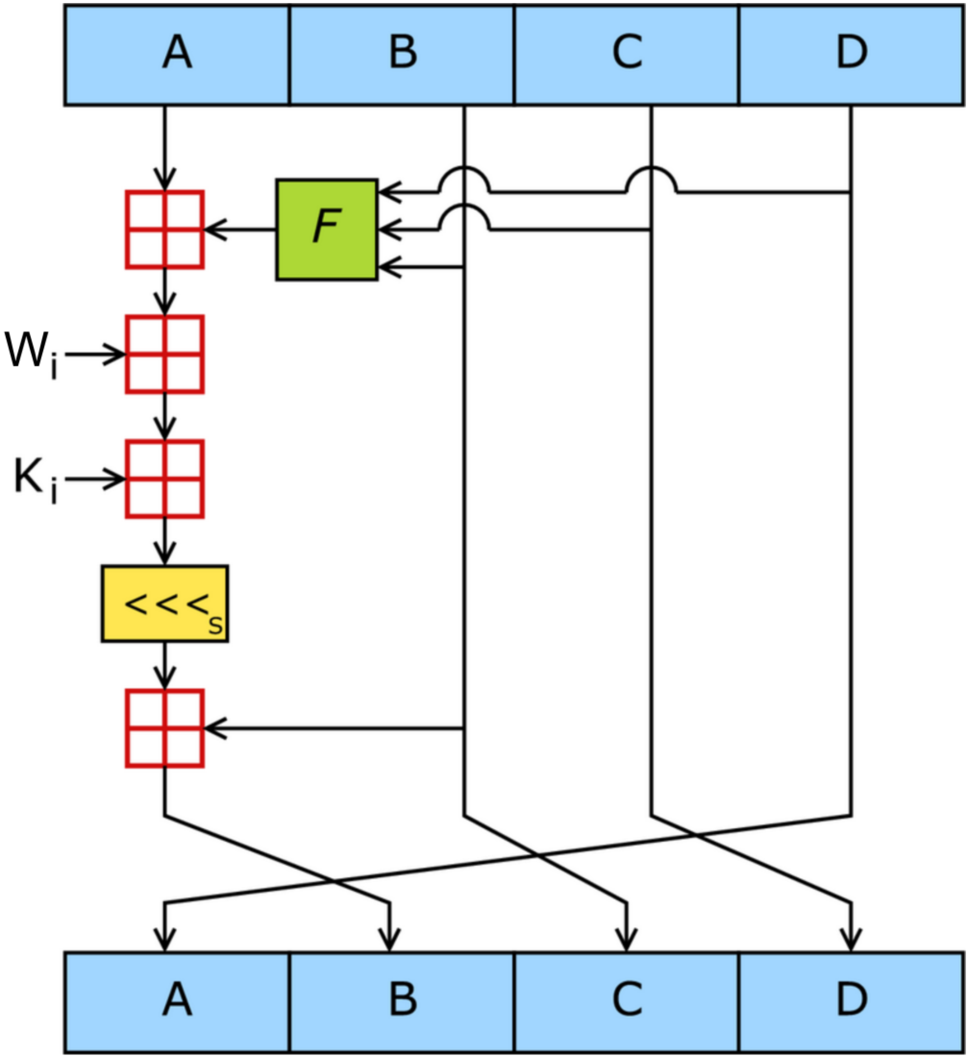
  $f_i(x,y,z) = xz \text{ OR } y(\neg z)$ for $i = 17, \ldots, 32$,

  $f_i(x,y,z) = x \text{ XOR } y \text{ XOR } z$ for $i = 33, \ldots, 48$,

  $f_i(x,y,z) = y \text{ XOR } (y \text{ OR } (\neg z))$ for $i = 49, \ldots, 64$,

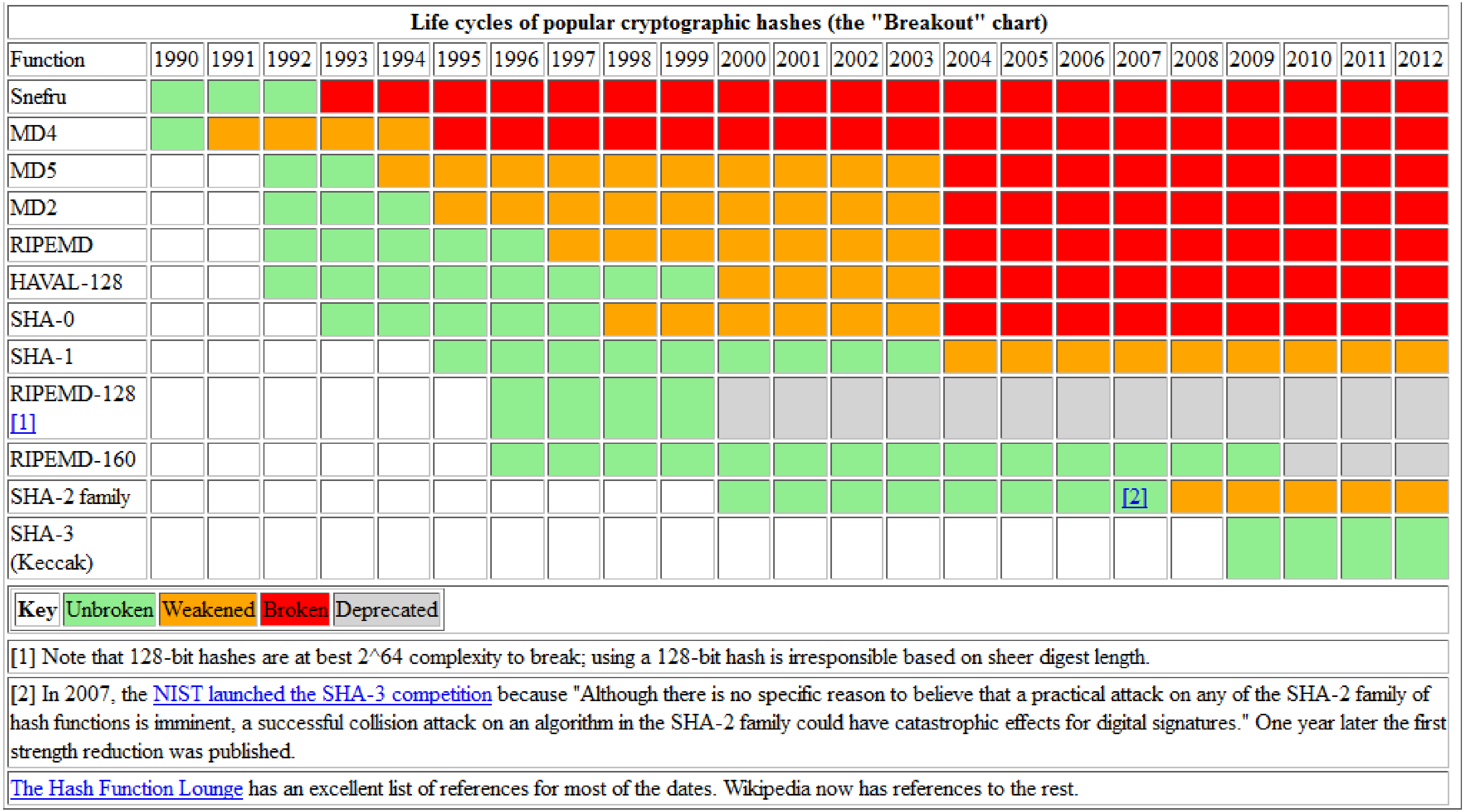
  these functions are nonlinear, balanced, and have an *avalanche effect*

step operations in MD5



provable hash functions

- people don’t like that one can’t prove much about hash functions
- reduction to established ‘hard problem’ such as factoring is seen as an advantage
- Example: VSH – Very Smooth Hash
  - Contini-Lenstra-Steinfeld 2006
  - collision resistance provable under assumption that a problem directly related to factoring is hard
  - but still far from ideal
    - bad performance compared to SHA-256
    - all kinds of multiplicative relations between hash values exist
    - not post-quantum secure



Life cycles of popular cryptographic hashes (the "Breakout" chart)

Function	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Snefru	🟩	🟩	🟩	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥
MD4	🟩	🟧	🟧	🟧	🟧	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥
MD5			🟩	🟩	🟧	🟧	🟧	🟧	🟧	🟧	🟧	🟧	🟧	🟧	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥
MD2			🟩	🟩	🟩	🟧	🟧	🟧	🟧	🟧	🟧	🟧	🟧	🟧	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥
RIPEMD			🟩	🟩	🟩	🟩	🟩	🟧	🟧	🟧	🟧	🟧	🟧	🟧	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥
HAVAL-128			🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟧	🟧	🟧	🟧	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥
SHA-0				🟩	🟩	🟩	🟩	🟩	🟧	🟧	🟧	🟧	🟧	🟧	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥	🟥
SHA-1						🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟧	🟧	🟧	🟧	🟧	🟧	🟧	🟧	🟧
RIPEMD-128 [1]							🟩	🟩	🟩	🟩	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨	▨
RIPEMD-160							🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	▨	▨	▨
SHA-2 family											🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩 [2]	🟧	🟧	🟧	🟧	🟧
SHA-3 (Keccak)																				🟩	🟩	🟩	🟩

Key	🟩 Unbroken	🟧 Weakened	🟥 Broken	▨ Deprecated

[1] Note that 128-bit hashes are at best 2^64 complexity to break; using a 128-bit hash is irresponsible based on sheer digest length.

[2] In 2007, the NIST launched the SHA-3 competition because "Although there is no specific reason to believe that a practical attack on any of the SHA-2 family of hash functions is imminent, a successful collision attack on an algorithm in the SHA-2 family could have catastrophic effects for digital signatures." One year later the first strength reduction was published.

The Hash Function Lounge has an excellent list of references for most of the dates. Wikipedia now has references to the rest.

SHAttered

The first concrete collision attack against SHA-1
https://shattered.io

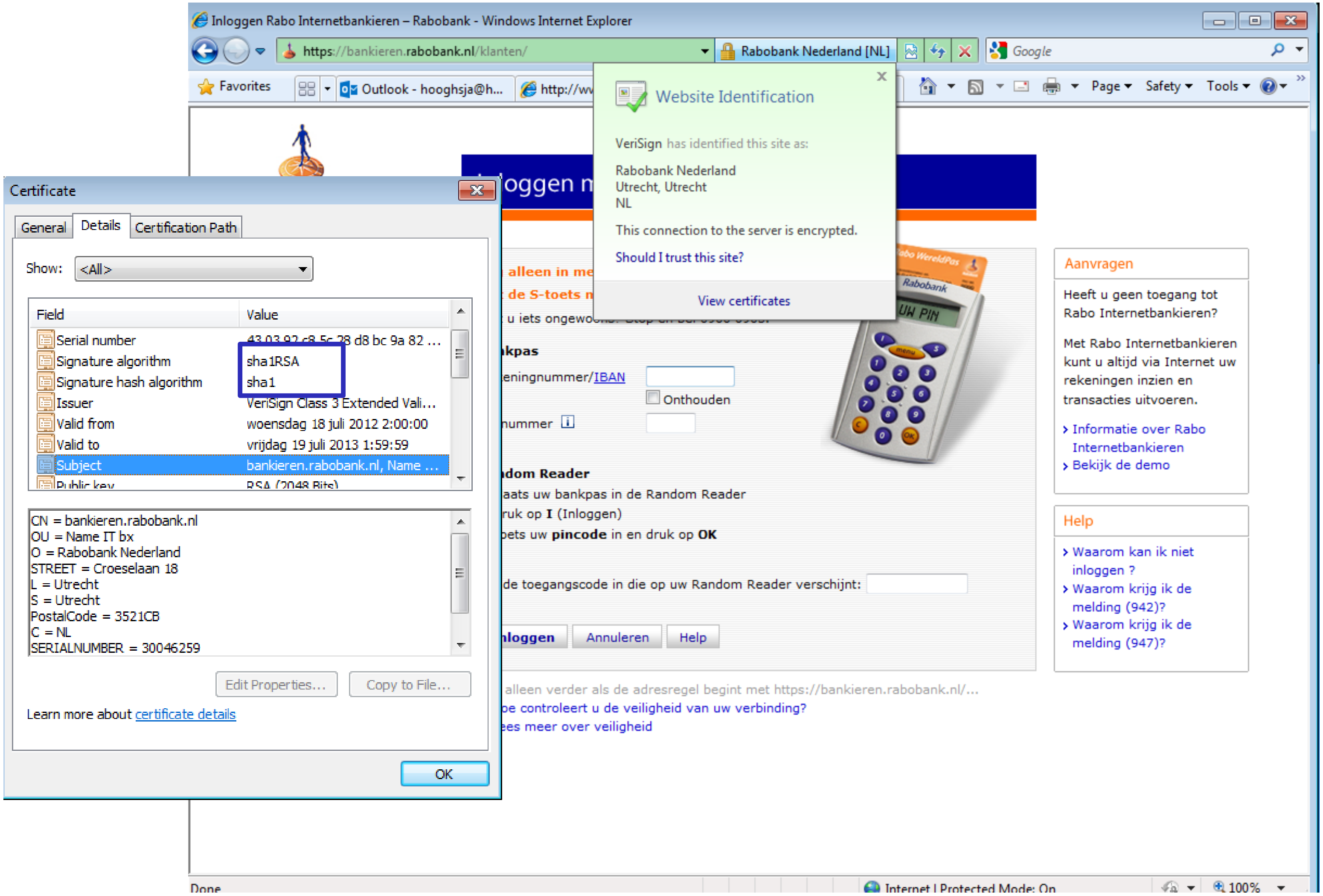
SHAttered

The first concrete collision attack against SHA-1
https://shattered.io

```
└─ sha1sum *.pdf
38762cf7f55934b34d179ae6a4c80cadccbb7f0a  1.pdf
38762cf7f55934b34d179ae6a4c80cadccbb7f0a  2.pdf
┌─ 🗁 /tmp/sha1                                                  0.64G   8-11h
└─ sha256sum *.pdf
2bb787a73e37352f92383abe7e2902936d1059ad9f1ba6daaa9c1e58ee6970d0  1.pdf
d4488775d29bdef7993367d541064dbdda50d383f89f0aa13a6ff2e0894ba5ff  2.pdf
```

Real life attacks on MD5

Example Hash-then-Sign in Browser



Inloggen Rabo Internetbankieren – Rabobank - Windows Internet Explorer

https://bankieren.rabobank.nl/klanten/

Rabobank Nederland [NL]

Website Identification

VeriSign has identified this site as:

Rabobank Nederland
Utrecht, Utrecht
NL

This connection to the server is encrypted.

Should I trust this site?

View certificates

Certificate

Show: <All>

Field	Value
Serial number	43 03 92 c8 5c 28 d8 bc 9a 82 …
Signature algorithm	sha1RSA
Signature hash algorithm	sha1
Issuer	VeriSign Class 3 Extended Vali…
Valid from	woensdag 18 juli 2012 2:00:00
Valid to	vrijdag 19 juli 2013 1:59:59
Subject	bankieren.rabobank.nl, Name …
Public key	RSA (2048 Bits)

CN = bankieren.rabobank.nl
OU = Name IT bx
O = Rabobank Nederland
STREET = Croeselaan 18
L = Utrecht
S = Utrecht
PostalCode = 3521CB
C = NL
SERIALNUMBER = 30046259

Learn more about certificate details

alleen in me
de S-toets n
u iets ongewo

kpas
eningnummer/IBAN
☐ Onthouden
nummer

dom Reader
aats uw bankpas in de Random Reader
ruk op **I** (Inloggen)
oets uw **pincode** in en druk op **OK**

de toegangscode in die op uw Random Reader verschijnt:

alleen verder als de adresregel begint met https://bankieren.rabobank.nl/…
oe controleert u de veiligheid van uw verbinding?
ees meer over veiligheid

Aanvragen

Heeft u geen toegang tot Rabo Internetbankieren?

Met Rabo Internetbankieren kunt u altijd via Internet uw rekeningen inzien en transacties uitvoeren.

- Informatie over Rabo Internetbankieren
- Bekijk de demo

Help

- Waarom kan ik niet inloggen ?
- Waarom krijg ik de melding (942)?
- Waarom krijg ik de melding (947)?

Done

Internet | Protected Mode: On

Wang’s attack on MD5

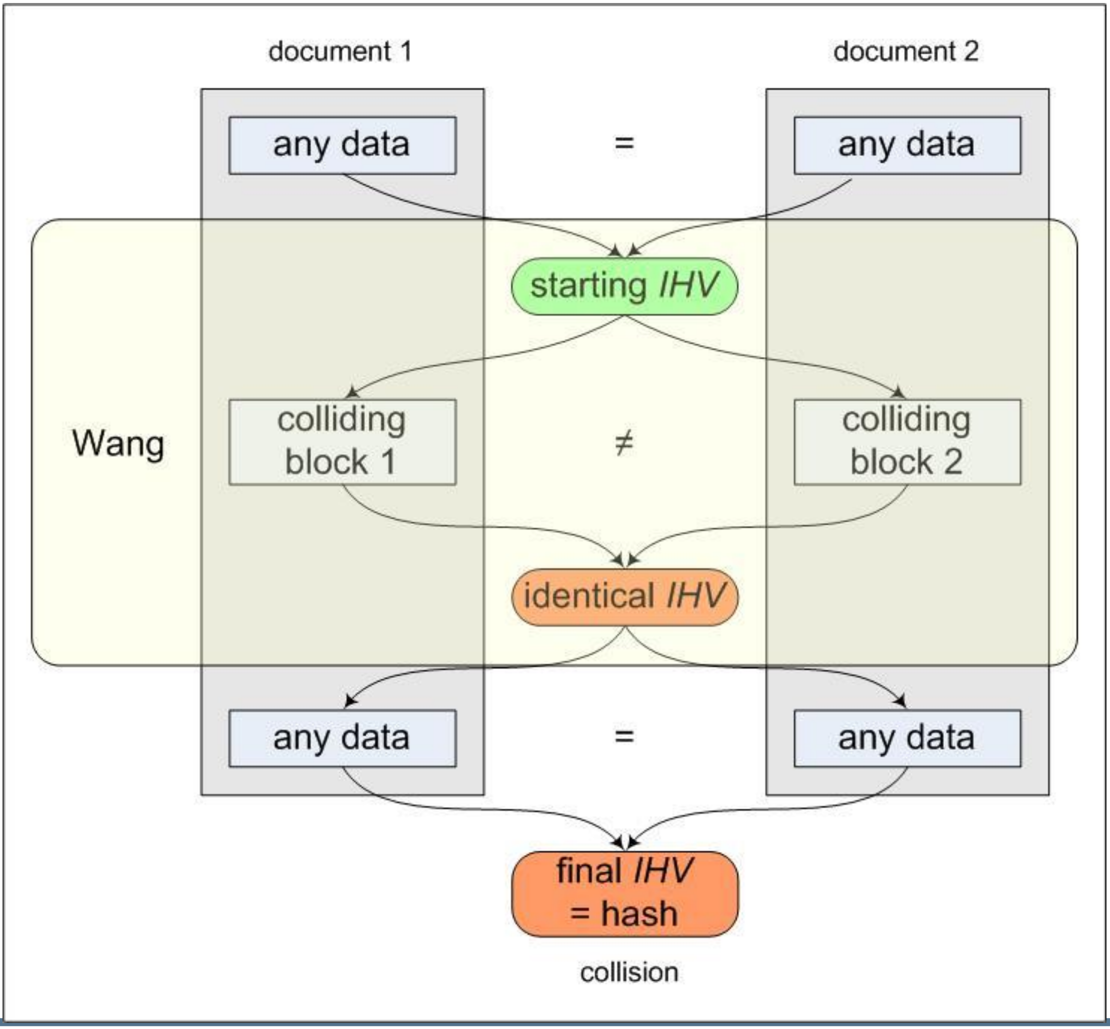
- *two-block* collision
  - for any input *IHV*, identical for the two messages i.e. $IHV_0 = IHV_0'$, $\Delta IHV_0 = 0$
  - *near-collision* after first block: $IHV_1 = CF(IHV_0,m_1)$, $IHV_1' = CF(IHV_0,m_1')$, with ΔIHV_1 having only a few carefully chosen ±1s
  - full collision after second block: $IHV_2 = CF(IHV_1,m_2)$, $= CF(IHV_1',m_2')$, i.e. $IHV_2 = IHV_2'$, $\Delta IHV_2 = 0$
- with IHV_0 the standard *IV* for MD5, and a third block for padding and MD-strengthening, this gives a collision for the full MD5

chosen-prefix collisions

- **latest development on MD5**
- **Marc Stevens (TU/e MSc student) 2006**
  - paper by Marc Stevens, Arjen Lenstra and Benne de Weger, EuroCrypt 2007
- **Marc Stevens (CWI PhD student) 2009**
  - paper by Marc Stevens, Alex Sotirov, Jacob Appelbaum, David Molnar, Dag Arne Osvik, Arjen Lenstra and Benne de Weger, Crypto 2007
  - rogue CA attack

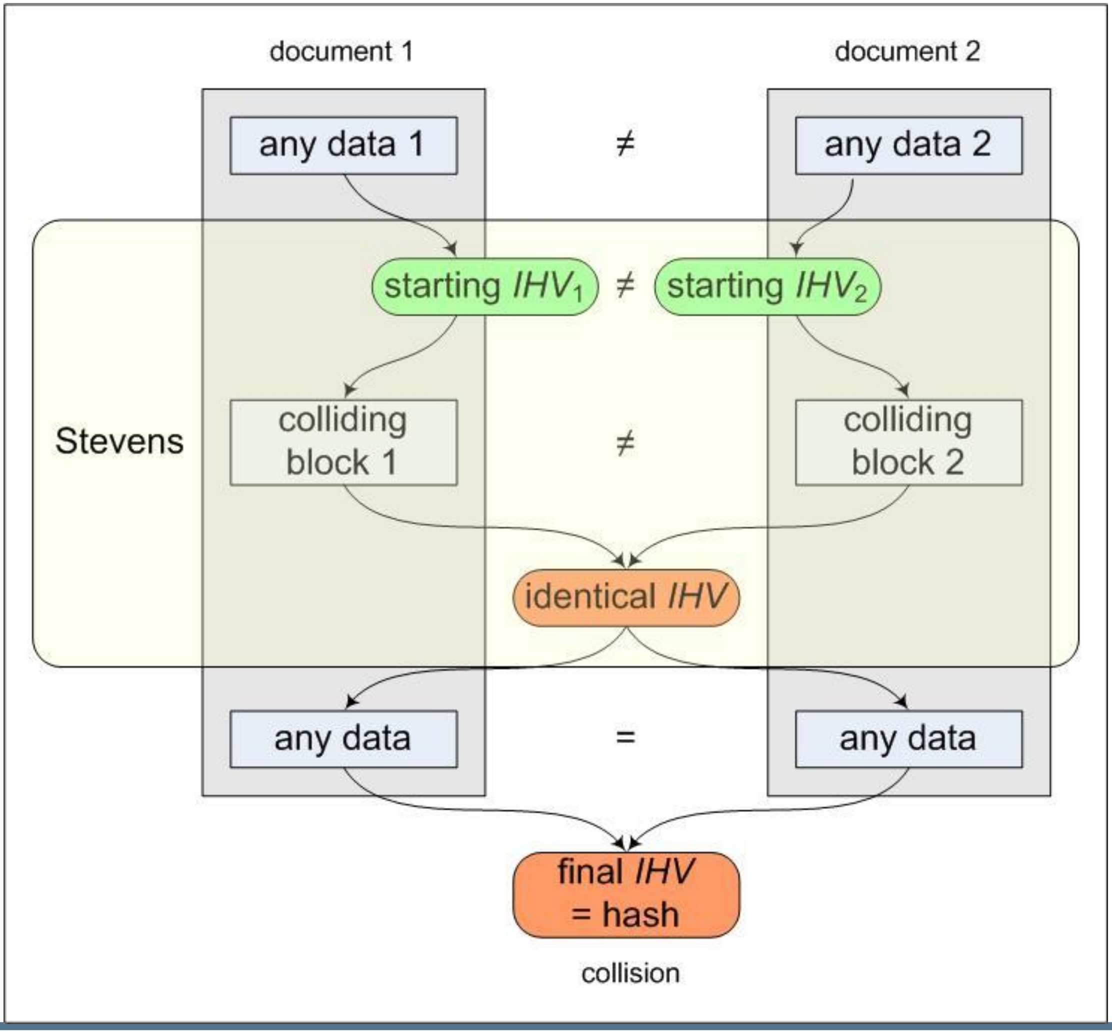
MD5: identical IV attacks

- **all attacks following Wang's method, up to recently**
- **MD5 collision attacks work for any starting *IHV***
  - data before and after the collision can be *chosen at will*
- **but starting *IHV*s must be identical**
  - data before and after the collision *must be identical*
- **called *random collision***



MD5: different IV attacks

- **new attack**
  - **Marc Stevens, TU/e**
  - **Oct. 2006**
- **MD5 collisions for any starting pair {*IHV_1*, *IHV_2*}**
  - **data before the collision *needs not to be identical***
  - **data before the collision can still be chosen at will, for each of the two documents**
  - **data after the collision still must be identical**
- **called *chosen-prefix collision***



indeed that was not the end in 2008 the ethical hackers came by

observation: commercial certification authorities still use MD5

idea: proof of concept of realistic attack as wake up call

→ attack a real, commercial certification authority

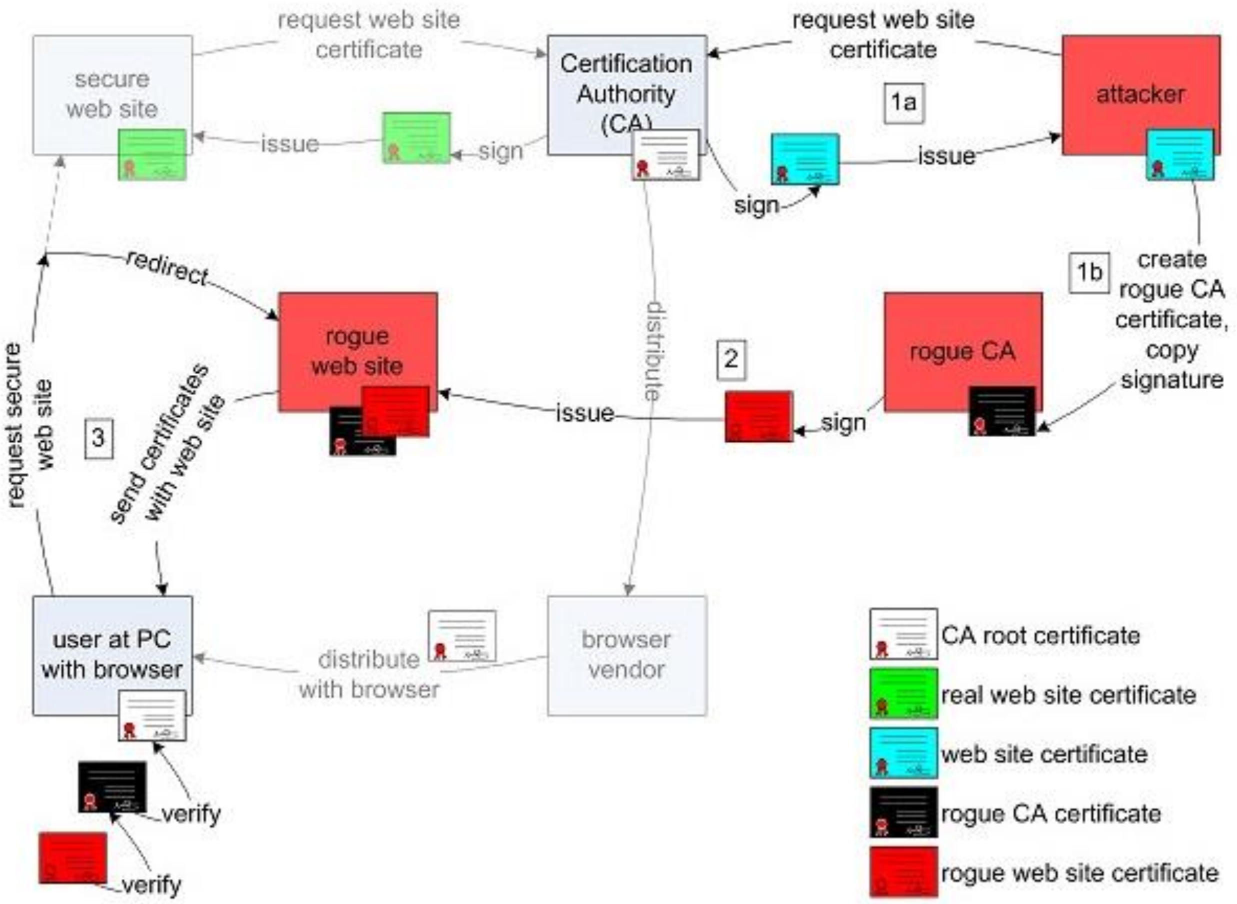
purchase a web certificate for a valid web domain

but with a “little tweak” built in

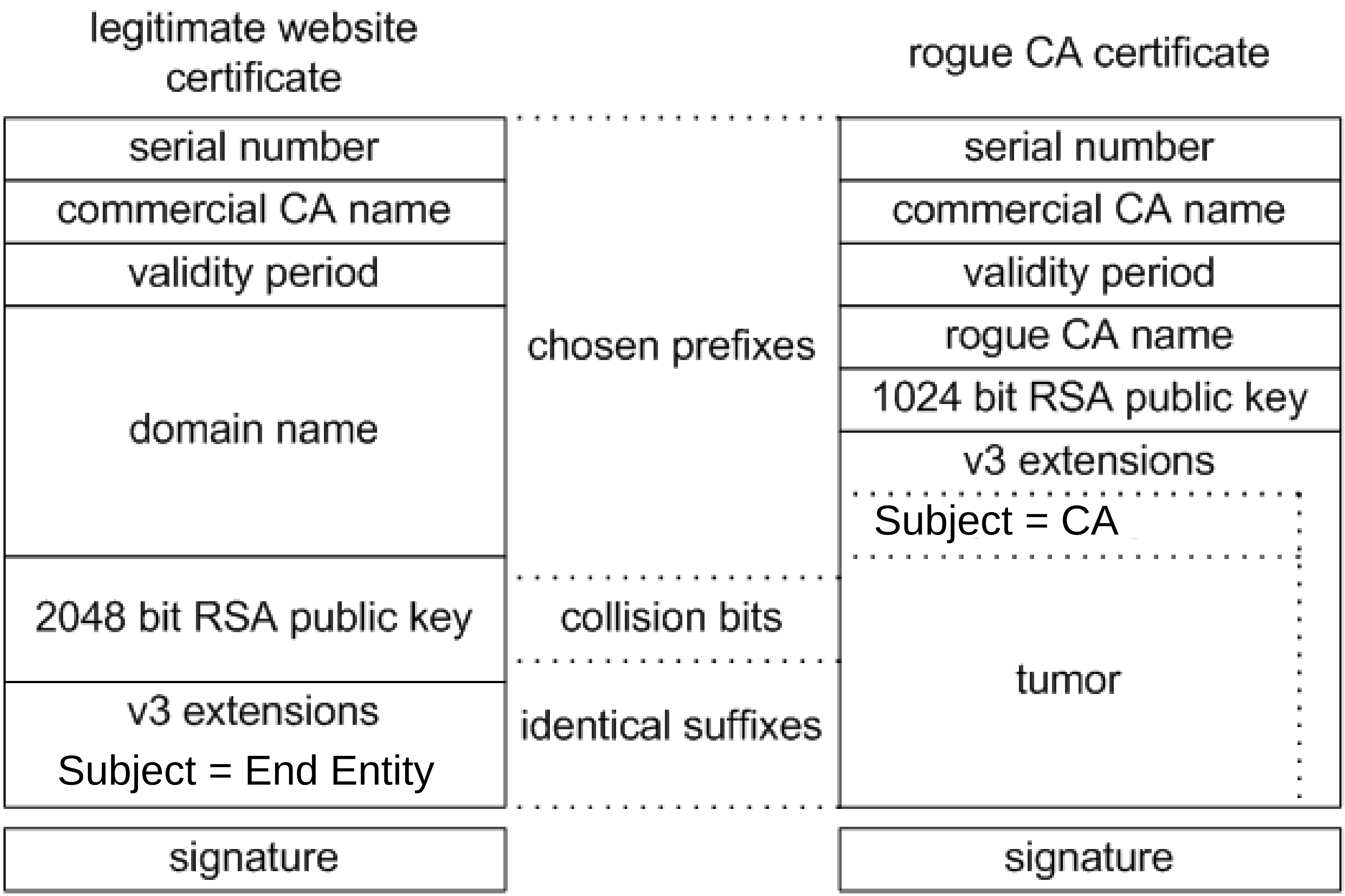
prepare a rogue CA certificate with identical MD5 hash

the commercial CA’s signature also holds for the rogue CA certificate

Outline of the RogueCA Attack



colliding certificates using chosen-prefix collisions, 2008



legitimate website certificate

- serial number
- commercial CA name
- validity period
- domain name
- 2048 bit RSA public key
- v3 extensions
  Subject = End Entity
- signature

chosen prefixes

collision bits

identical suffixes

rogue CA certificate

- serial number
- commercial CA name
- validity period
- rogue CA name
- 1024 bit RSA public key
- v3 extensions
  Subject = CA
- tumor
- signature

problems to be solved

- predict the serial number
- predict the time interval of validity
    - at the same time
    - a few days before
- more complicated certificate structure
    - “Subject Type” after the public key
- small space for the collision blocks
    - is possible but much more computations needed
- not much time to do computations
    - to keep probability of prediction success reasonable

how difficult is predicting?

time interval:

CA uses automated certification procedure
certificate issued exactly 6 seconds after click

serial number :

Nov 3 07:44:08 2008 GMT	643006
Nov 3 07:45:02 2008 GMT	643007
Nov 3 07:46:02 2008 GMT	643008
Nov 3 07:47:03 2008 GMT	643009
Nov 3 07:48:02 2008 GMT	643010
Nov 3 07:49:02 2008 GMT	643011
Nov 3 07:50:02 2008 GMT	643012
Nov 3 07:51:12 2008 GMT	643013
Nov 3 07:51:29 2008 GMT	643014
Nov 3 07:52:02 2008 GMT	have a guess…

the attack at work

estimated: 800-1000 certificates issued in a weekend

procedure:

1. buy certificate on Friday, serial number *S*-1000
2. predict serial number *S* for time *T* Sunday evening
3. make collision for serial number *S* and time *T*: 2 days time
4. short before *T* buy additional certificates until *S*-1
5. buy certificate on time *T-6*
   hope that nobody comes in between and steals our serial number *S*

to let it work

cluster of >200
PlayStation3
game consoles
(1 PS3 = 40 PC’s)

complexity: 2^{50}
memory: 30 GB

→ collision in 1 day

result

success after 4th attempt (4th weekend)

purchased a few hundred certificates

- (promotion action: 20 for one price)
- total cost: < US$ 1000

conclusion on MD5

- **at this moment, ‘meaningful’ hash collisions are**
  - easy to make
  - but also easy to detect
  - still hard to abuse realistically
- **with chosen-prefix collisions we come close to realistic attacks**
- **to do *real* harm, second pre-image attack needed**
  - real harm is e.g. forging digital signatures
  - this is not possible yet, not even with MD5
- **More information: http://www.win.tue.nl/hashclash/**

Questions?

proof of birthday paradox

- probability that all k elements are distinct is

$$\prod_{i=0}^{k-1} \frac{t-i}{t} = \prod_{i=0}^{k-1}\left(1-\frac{i}{t}\right) \le \prod_{i=0}^{k-1} e^{-\frac{i}{t}} = e^{-\sum_{i=0}^{k-1}\frac{i}{t}} = e^{-\frac{k(k-1)}{2t}}$$

and this is $< ½$ when $k(k\text{-}1) > (2 \log 2)t$

$(\approx k^2)$ $(\approx 1.4\, t)$