Post-Quantum Cryptography & Privacy

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PQCRYPTO
ICT-645622

Privacy?


... the Panopticon must not be understood as a dream building: it is the diagram of a mechanism of power reduced to its ideal form.
Michel Foucault, *Discipline and Punish*, 1977

Too abstract?







How to achieve privacy?

Tor

DuckDuckGo

Under the hood…

Public-key crypto

- ECC
- RSA
- DSA

Secret-key crypto

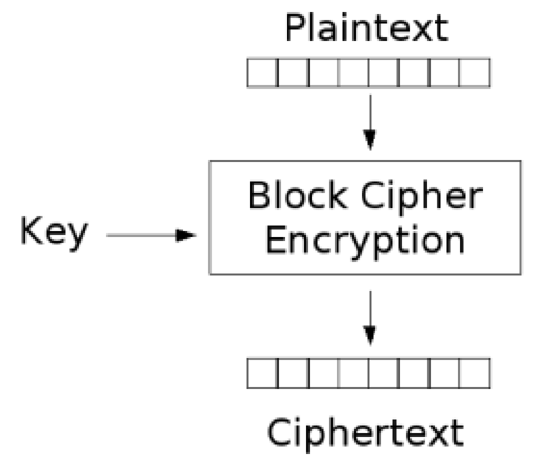
- AES
- SHA2
- SHA1
- …

Combination of both needed!

Secret-key cryptography

Main (Secret-key) primitives

- Block- / Stream Cipher
  - Encryption of data
  - Provides Secrecy
- Massage authentication code
  - Authentication of data
  - Provides authenticity
- Hash function
  - Cryptographic checksum
  - Allows efficient comparison



Public-key cryptography

Main (public-key) primitives

- Digital signature
  - Proof of authorship
  - Provides:
    - Authentication
    - Non-repudiation

- Public-key encryption / key exchange
  - Establishment of commonly known secret key
  - Provides secrecy

Applications

- Code signing (Signatures)
  - Software updates
  - Software distribution
  - Mobile code
- Communication security (Signatures, PKE / KEX)
  - TLS, SSH, IPSec, ...
  - eCommerce, online banking, eGovernment, ...
  - Private online communication

Connection security (simplified)

Hi

pk, Cert(pk belongs to shop)

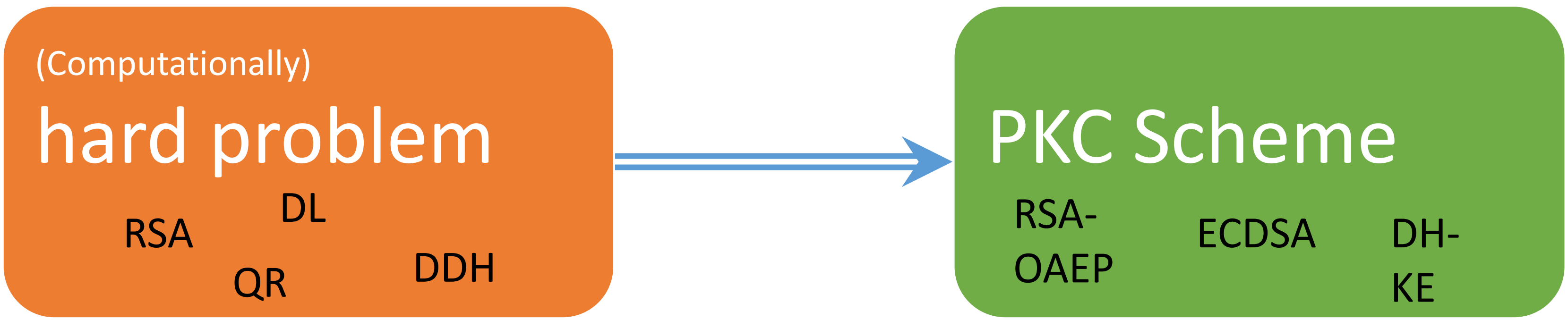
PKC to establish shared secret sk

SKC secured communication using sk

The TEA SHOP FINE TEAS & TISANES PURE & BLENDED

We need secret- and public-key crypto to achieve privacy!

How to build PKC



Quantum Computing

Quantum Computing

“Quantum computing studies theoretical computation systems (quantum computers) that make direct use of quantum-mechanical phenomena, such as superposition and entanglement, to perform operations on data.”

-- Wikipedia

Qubits

- Qubit state: $\alpha_0\,|0\rangle + \alpha_1\,|1\rangle$ with $\alpha_i \in \mathbb{C}$ such that $|\alpha_0|^2 + |\alpha_1|^2 = 1$
- Ket: $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}, |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$
- Qubit can be in state $\frac{|0\rangle+|1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

- Computing with 0 and 1 at the same time!

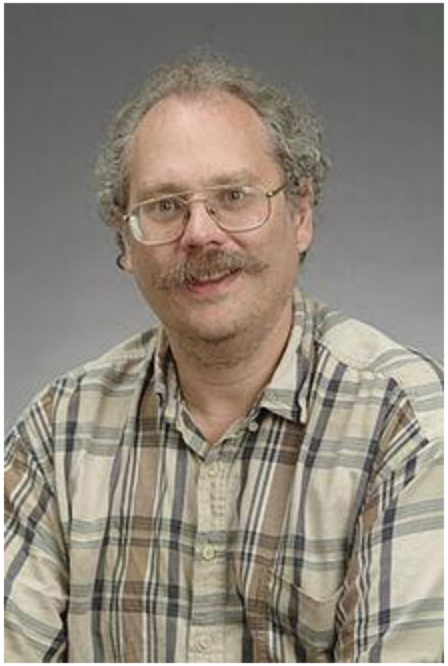
Quantum computers are not almighty

- To learn outcome one has to measure.
  - Collapses state
  - 1 qubit leads 1 classical bit of information
  - Randomized process
- Only invertible computation.
- Impossible to clone (copy) quantum state.

The Quantum Threat

Shor‘s algorithm (1994)

- Quantum computers can do FFT very efficiently
- Can be used to find period of a function
- This can be exploited to factor efficiently (RSA)
- Shor also shows how to solve discrete log efficiently (DSA, DH, ECDSA, ECDH)



Grover‘s algorithm (1996)

- Quantum computers can search N entry DB in $\Theta(\sqrt{N})$
- Application to symmetric crypto
- Nice: Grover is provably optimal (For random function)
- Double security parameter.

To sum up

- All asymmetric crypto is broken by QC
  - No more digital signatures
  - No more public key encryption
  - No more key exchange

- No secure shopping for tea...

Quantum Cryptography

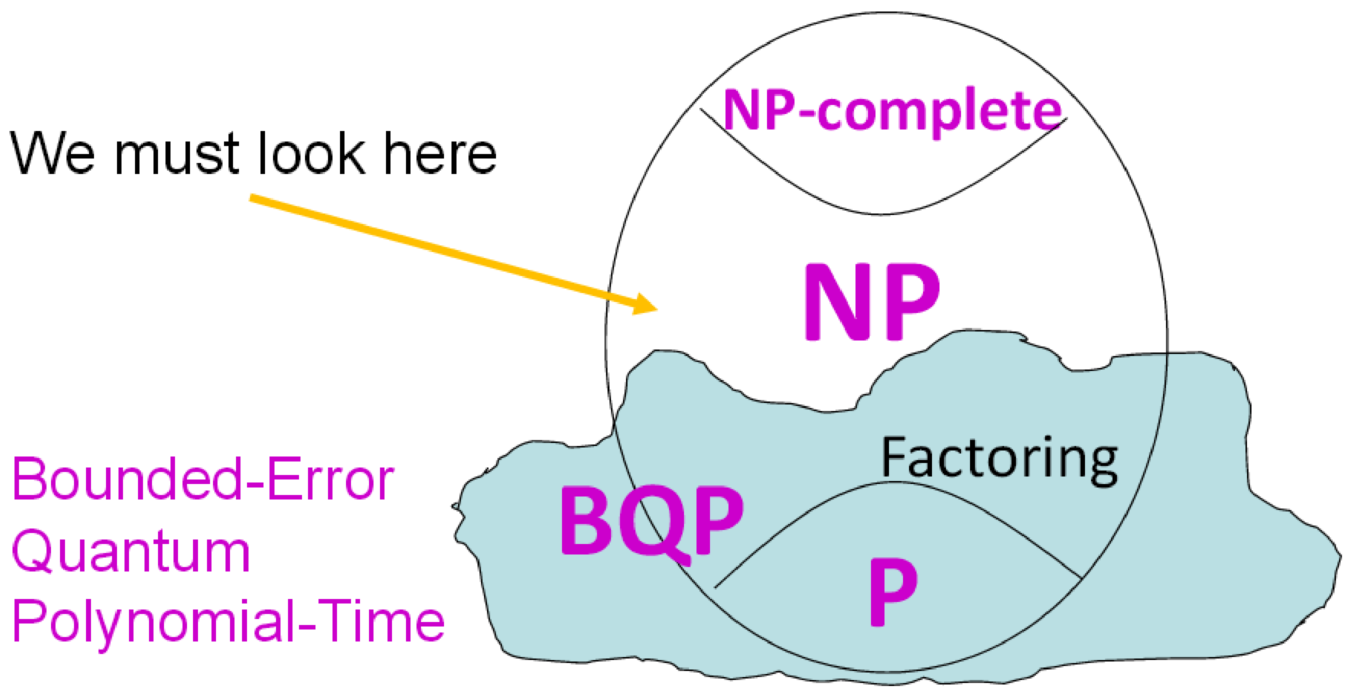
Why not beat ‘em with their own weapons?

- QKD: Quantum Key distribution.
  - Based on some nice quantum properties: entanglement & collapsing measurments
  - Information theoretic security (at least in theory) -> Great!
  - For sale today!
- So why don‘t we use this?
- Only short distance, point-to-point connections!
  - Internet? No way!
- Longer distances require „trusted-repeaters“ ☺
  - We all know where this leads...

PQCRYPTO to the rescue

Quantum-secure problems

No provably quantum resistant problems



Credits: Buchmann, Bindel 2015

Conjectured quantum-secure problems

- Solving multivariate quadratic equations (MQ-problem)
  -> Multivariate Crypto
- Bounded-distance decoding (BDD)
  -> Code-based crypto
- Short(est) and close(st) vector problem (SVP, CVP)
  -> Lattice-based crypto
- Breaking security of symmetric primitives (SHAx-, AES-, Keccak-,... problem)
  -> Hash-based signatures / symmetric crypto

MQ-Problem

Let $\boldsymbol{x} = (x_1, \dots, x_n) \in \mathbb{F}_q^n$ and $\mathbf{MQ}(n, m, \mathbb{F}_q)$ denote the family of vectorial functions $\boldsymbol{F}: \mathbb{F}_q^n \longrightarrow \mathbb{F}_q^m$ of degree 2 over $\mathbb{F}_q$:

$\mathbf{MQ}(n, m, \mathbb{F}_q)$

$$= \left\{ \boldsymbol{F}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), \dots, f_m(\boldsymbol{x}) | f_s(\boldsymbol{x}) = \sum_{i,j} a_{i,j} x_i x_j + \sum_i b_i x_i \,, \qquad s \in [1, m] \right\}$$

The $\mathbf{MQ}$ Problem $\mathbf{MQ}(\boldsymbol{F}, \boldsymbol{v})$ is defined as given $\boldsymbol{v} \in \mathbb{F}_q^m$ find, if any, $\boldsymbol{s} \in \mathbb{F}_q^n$ such that $\boldsymbol{F}(\boldsymbol{s}) = \boldsymbol{v}$.

Decisional version is NP-complete [Garey, Johnson´79]

Multivariate Signatures
(trad. approach)

$\mathrm{P}: \mathrm{F}^{\mathrm{n}} \rightarrow \mathrm{F}^{\mathrm{m}}$, easily invertible non-linear

$\mathrm{S}: \mathrm{F}^{\mathrm{n}} \rightarrow \mathrm{F}^{\mathrm{n}}$, $\mathrm{T}: \mathrm{F}^{\mathrm{m}} \rightarrow \mathrm{F}^{\mathrm{m}}$, affine linear

Public key: $\mathrm{G} = \mathrm{S} \circ \mathrm{P} \circ \mathrm{T}$, hard to invert

Secret Key: S, P, T allows to find G^{-1}

$$\mathrm{G}^{-1} = \mathrm{T}^{-1} \circ \mathrm{P}^{-1} \circ \mathrm{S}^{-1}$$

Signing: $\mathrm{s} = \mathrm{T}^{-1} \circ \mathrm{P}^{-1} \circ \mathrm{S}^{-1}(\mathrm{m})$

Verifying: $\mathrm{G}(\mathrm{s}) =^{?} \mathrm{m}$

Forging signature: Solve $\mathrm{G}(\mathrm{s}) - \mathrm{m} = 0$

Fast

Large keys:
100 kBit for 100 bit security
Compared to 1776 bit RSA modulus

- UOV , Goubin et al., 1999
- Rainbow, Ding, et al. 2005
- pFlash, Cheng, 2007
- Gui, Ding, Petzoldt, 2015

Credits: Buchmann, Bindel 2015

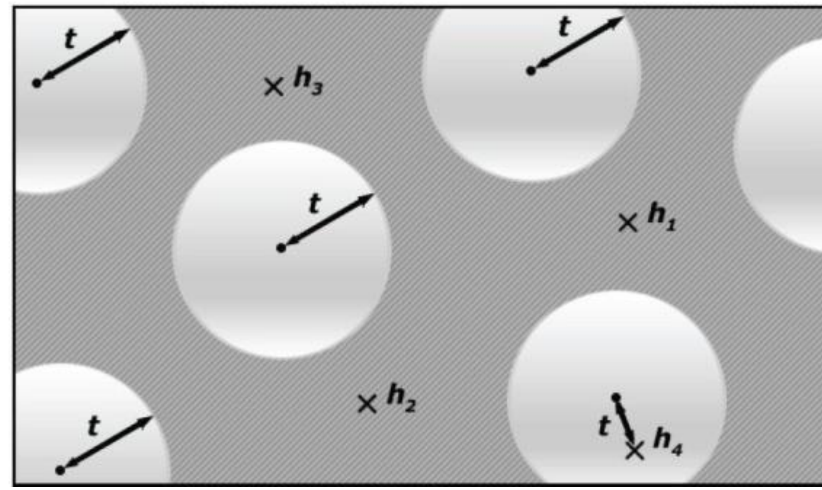
Multivariate Cryptography

- Breaking scheme $\not\Leftrightarrow$ Solving random MQ-instance
  -> NP-complete is a worst-case notion
  (there might be – and there are for MQ -- easy instances)
  -> Not a random instance
  Many broken proposals
  -> Oil-and-Vinegar, SFLASH, MQQ-Sig, (Enhanced) TTS, Enhanced STS.
  -> Security somewhat unclear
- Only signatures
  -> (new proposal for encryption exists but too recent)
- Really large keys
- **New proposal with security reduction, small keys, but large signatures.**

Coding-based cryptography - BDD

Given:
- Linear code $C \subseteq F_2^n$
- $y \in F_2^n$
- $t \in \mathbb{N}$

Find:
- $x \in C$: $\mathrm{dist}(x, y) \leq t$



BDD is NP-complete (Berlekamp et al. 1978) (Decisional version)

Credits: Buchmann, Bindel 2015

McEliece PKE (1978)

S, G, P matrices over F

G generator matrix for Goppa code ← Allows to solve BDD

Public key: $G' = S \circ G \circ P$, t

Secret Key: P, S, G

Encryption: $c = mG' + z \in F^n$

Decryption: $x = cP^{-1} = mSG + zP^{-1}$

solve BDD to get $y = mSG$

decode to obtain m

Fast

Large public keys!
500 kBits for 100 bit security
Compared to 1776 bit RSA modulus

IND-CPA secure version

Credits: Buchmann, Bindel 2015

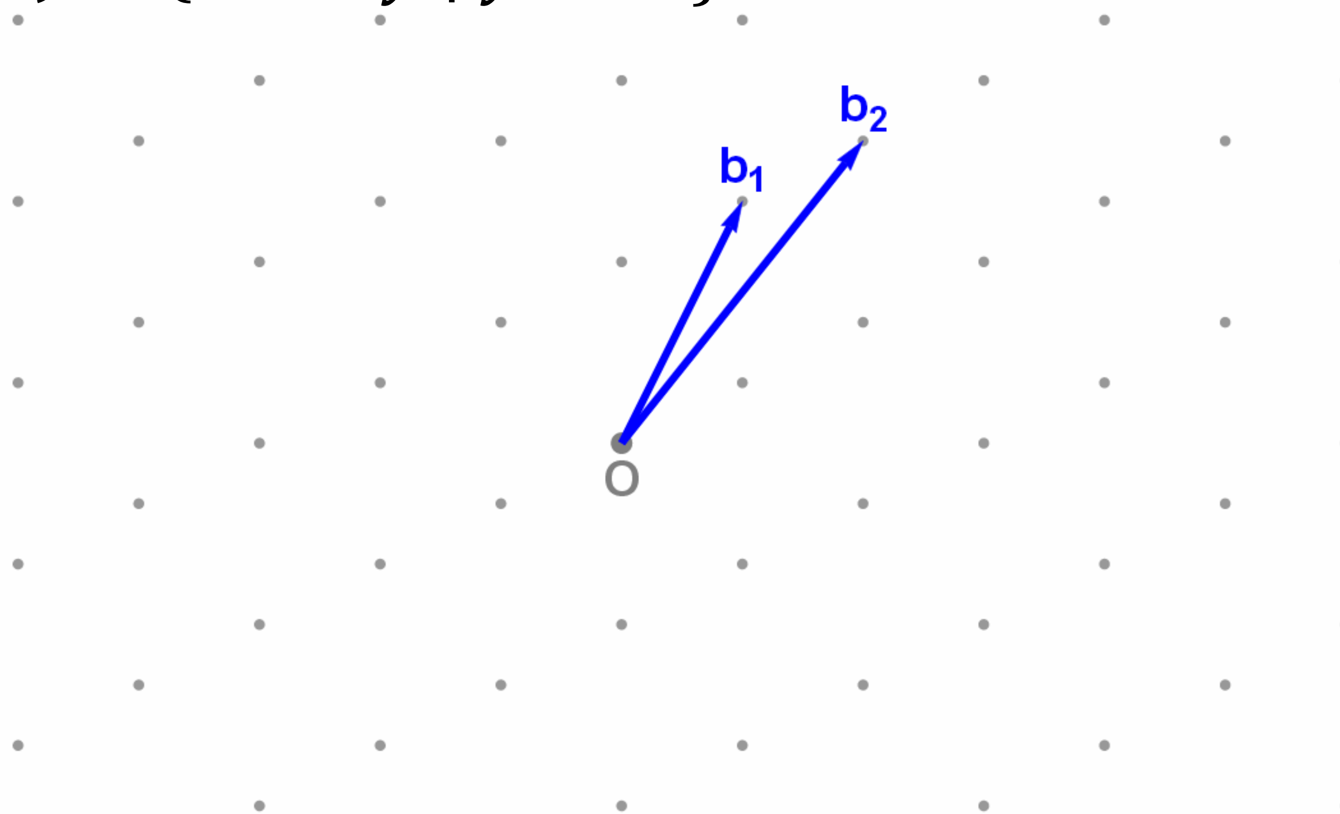
Code-based cryptography

- Breaking scheme $\not\Leftrightarrow$ Solving BDD
  -> NP-complete is a worst-case notion
  (there might be – and there are for BDD -- easy instances)
  -> Not a random instance
  However, McEliece with binary Goppa codes survived for almost 40 years (similar situation as for e.g. AES)
- Using more compact codes often leads to break
- So far, no practical signature scheme
- Really large public keys

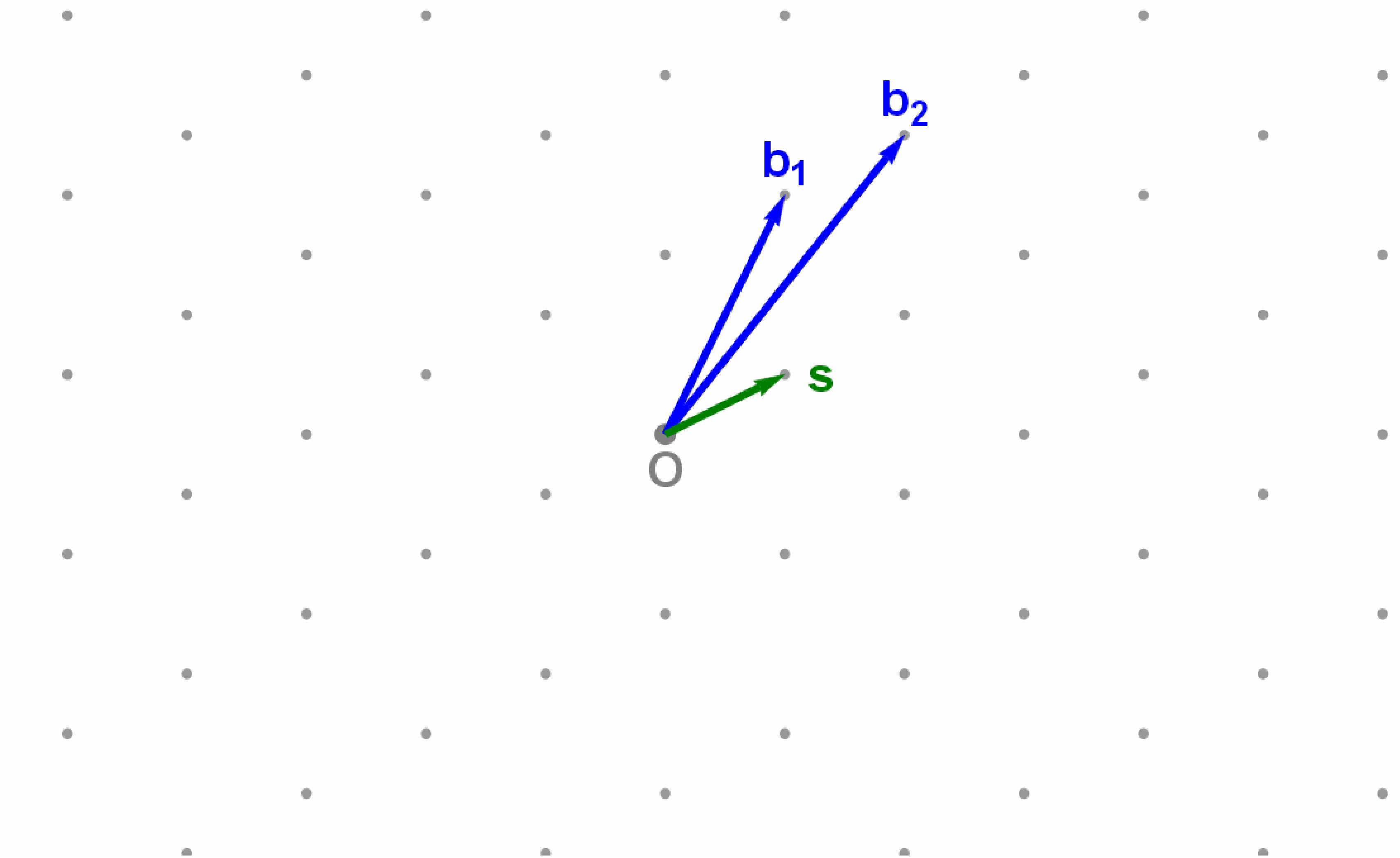
Lattice-based cryptography

Basis: $B = (b_1, b_2) \in \mathbb{Z}^{2\times 2}; b_1, b_2 \in \mathbb{Z}^2$

Lattice: $\Lambda(B) = \{x = By \mid y \in \mathbb{Z}^2\}$



Shortest vector problem (SVP)



(Worst-case) Lattice Problems

- **SVP:** Find shortest vector in lattice, given random basis. NP-hard (Ajtai'96)
- **Approximate SVP (αSVP):** Find short vector (norm < α times norm of shortest vector). Hardness depends on α (for α used in crypto not NP-hard).
- **CVP:** Given random point in underlying vectorspace (e.g. $\mathbb{Z}^n$), find the closest lattice point. (Generalization of SVP, reduction from SVP)
- **Approximate CVP (αCVP):** Find a „close" lattice point. (Generalization of αSVP)

(Average-case) Lattice Problems Short Integer Solution (SIS)

$\mathbb{Z}_p^n = \text{n-dim.}$ vectors with entries mod p ($\approx n^3$)

Goal:

Given $\boldsymbol{A} = (\boldsymbol{a_1}, \boldsymbol{a_2}, \dots, \boldsymbol{a_m}) \in \mathbb{Z}_p^{n \times m}$
Find „small“ $\boldsymbol{s} = (s_1, \dots, s_m) \in \mathbb{Z}^m$ such that

$$\boldsymbol{As} = \boldsymbol{0} \bmod p$$

Reduction from worst-case αSVP.

Hash function

Set $m > n \log p$ and define $f_{\boldsymbol{A}}: \{0,1\}^m \rightarrow \mathbb{Z}_p^n$ as

$$f_{\boldsymbol{A}}(\boldsymbol{x}) = \boldsymbol{A}\boldsymbol{x} \bmod p$$

Collision-resistance: Given short $\boldsymbol{x_1}, \boldsymbol{x_2}$ with $\boldsymbol{A}\boldsymbol{x_1} = \boldsymbol{A}\boldsymbol{x_2}$ we can find a short solution as

$$\boldsymbol{A}\boldsymbol{x_1} = \boldsymbol{A}\boldsymbol{x_2} \Rightarrow \boldsymbol{A}\boldsymbol{x_1} - \boldsymbol{A}\boldsymbol{x_2} = \boldsymbol{0}$$
$$\boldsymbol{A}(\boldsymbol{x_1} - \boldsymbol{x_2}) = \boldsymbol{0}$$

So, $\boldsymbol{z} = \boldsymbol{x_1} - \boldsymbol{x_2}$ is a solution and it is short as $\boldsymbol{x_1}, \boldsymbol{x_2}$ are short.

Lattice-based crypto

- SIS: Allows to construct signature schemes, hash functions, ... , basically minicrypt.
- For more advanced applications: Learning with errors (LWE)
  - Allows to build PKE, IBE, FHE,...
- Performance: Sizes can almost reach those of RSA (just small const. factor), really fast (for lattices defined using polynomials).
- BUT: Exact security not well accessed, yet. Especially, no good estimate for quantum computer aided attacks.

Hash-based Signature Schemes

[Mer89]

- Post quantum
- Only secure hash function
- Security well understood
- Fast

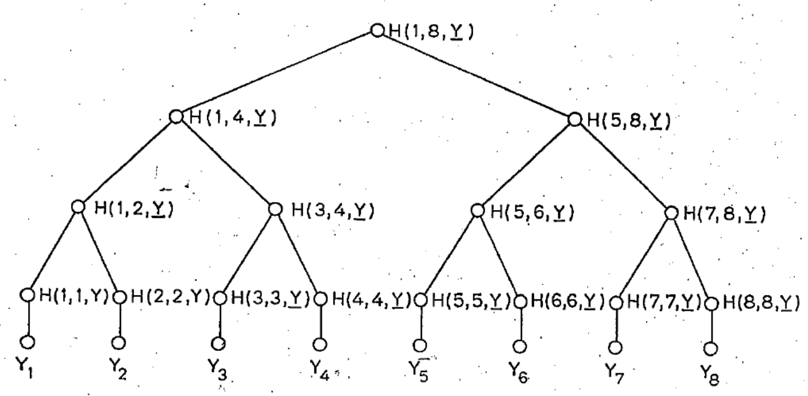


FIG 1

AN AUTHENTICATION TREE WITH N = 8.

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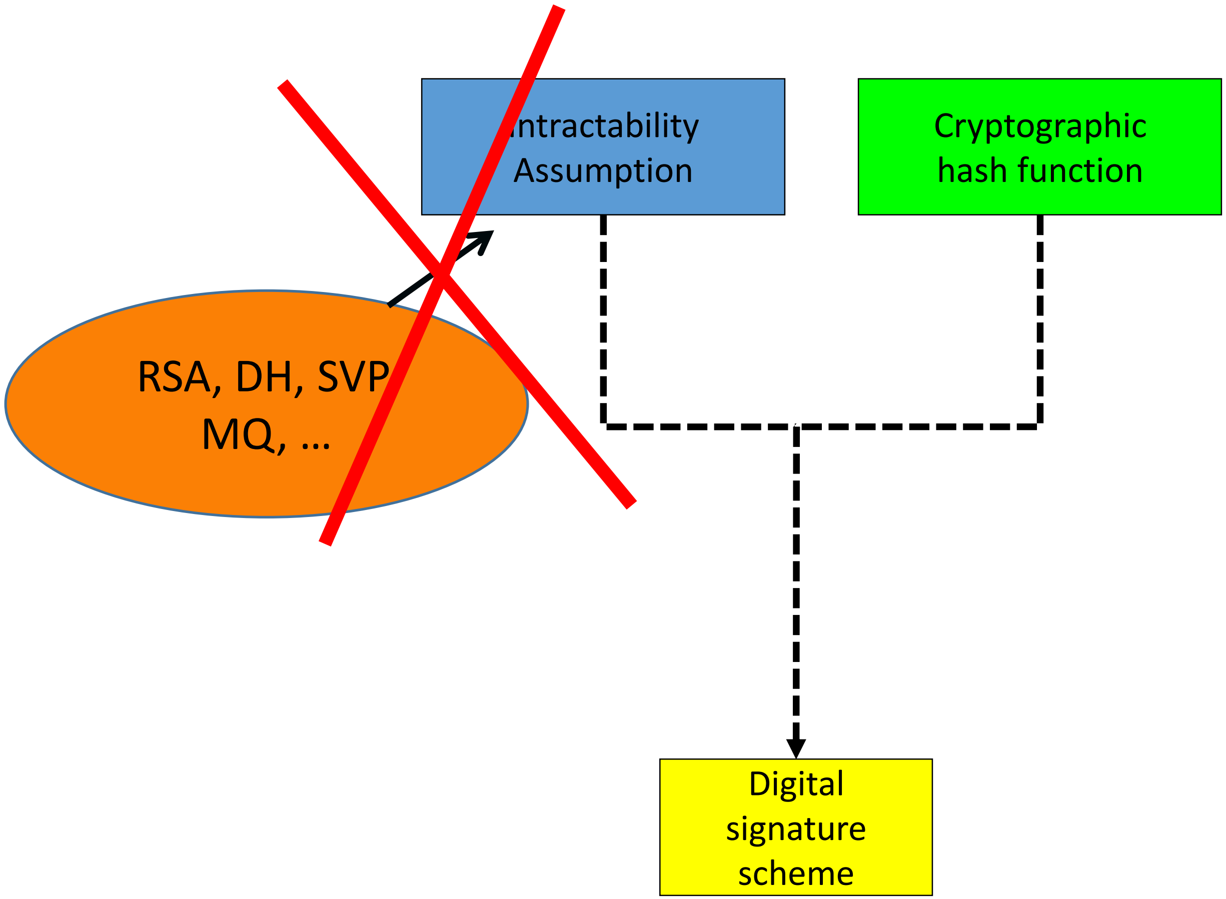
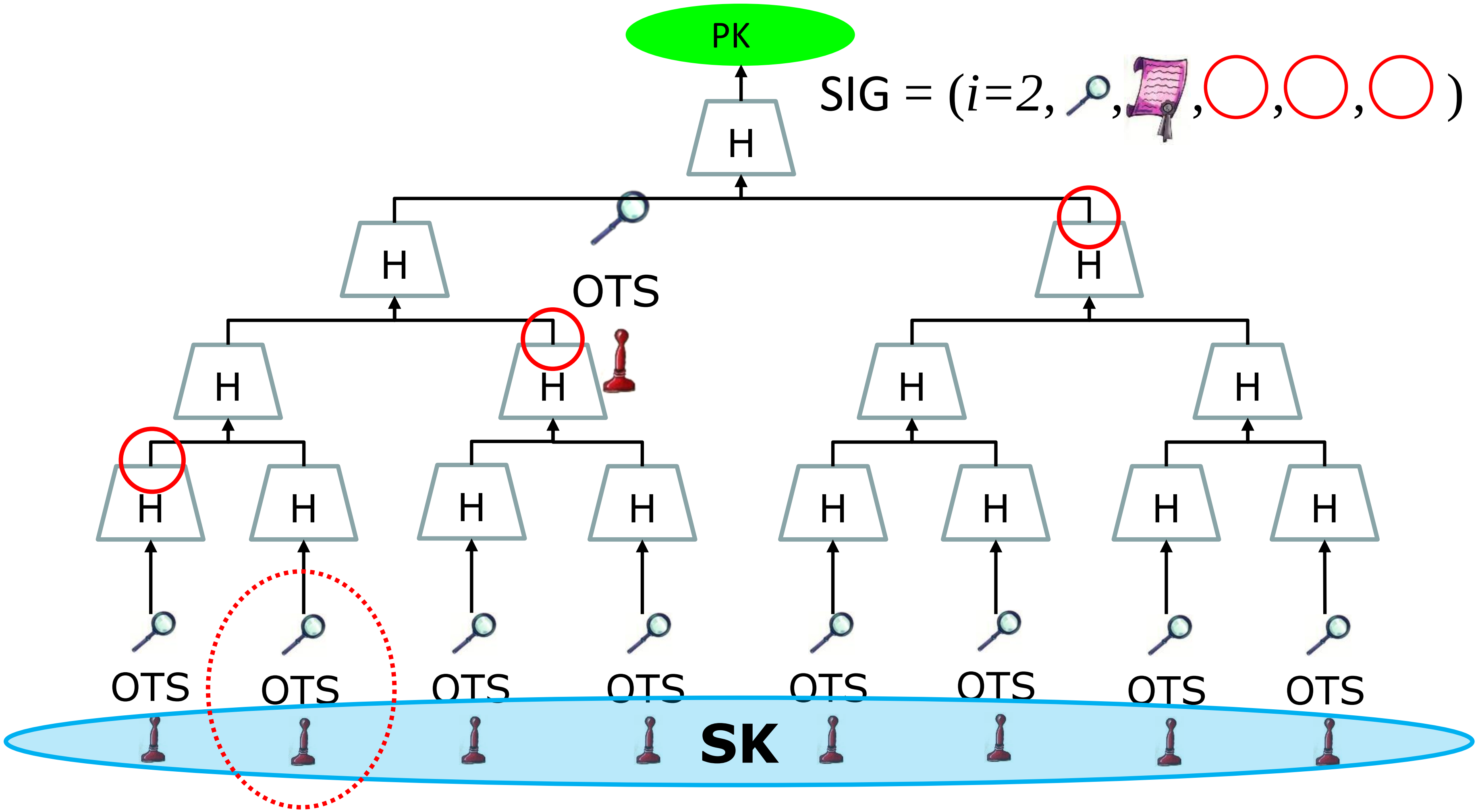
RSA – DSA – EC-DSA…



Merkle's Hash-based Signatures



Hash-based signatures

- Only signatures
- Minimal security assumptions
- Well understood
- Fast & compact (2kB, few ms), but stateful, or
- Stateless, bigger and slower (41kB, several ms).

- Two Internet drafts (drafts for RFCs), one in „RFC Editor queue“

NIST Competition


NIST National Institute of Standards and Technology
Information Technology Laboratory

SEARCH: Search

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POST-QUANTUM CRYPTO PROJECT

NEWS -- December 15, 2016: The National Institute of Standards and Technology (NIST) is now accepting submissions for quantum-resistant public-key cryptographic algorithms. The deadline for submission is ***November 30, 2017.*** Please see the Post-Quantum Cryptography Standardization menu at left for the complete submission requirements and evaluation criteria.

In recent years, there has been a substantial amount of research on quantum computers – machines that exploit quantum mechanical phenomena to solve mathematical problems that are difficult or intractable for conventional computers. If large-scale quantum computers are ever built, they will be able to break many of

Resources

- PQ Summer School: https://2017.pqcrypto.org/school/index.html
- NIST PQC Standardization Project: https://csrc.nist.gov/Projects/Post-Quantum-Cryptography
- Master Math (Selected Areas in Cryptology): https://elo.mastermath.nl/

PQCrypto

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Thank you!
Questions?