Post-Quantum Cryptography

a talk about problems... problems... problems

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The Problem

Public-key cryptography

Main (public-key) primitives

- Digital signature (DSIG)
  - Proof of authorship
  - Provides:
    - Authentication
    - Non-repudiation

- Public-key encryption (PKE) / Key exchange (KEX) / Key encapsulation mechanism (KEM)
  - Establishment of commonly known secret key
  - Provides secrecy

Applications

- Code signing (DSIG)
  - Software updates
  - Software distribution
  - Mobile code

- Communication security (DSIG, PKE / KEX /KEM)
  - TLS, SSH, IPSec, ...
  - eCommerce, online banking, eGovernment, ...
  - Private online communication

Connection security (simplified)

Hi

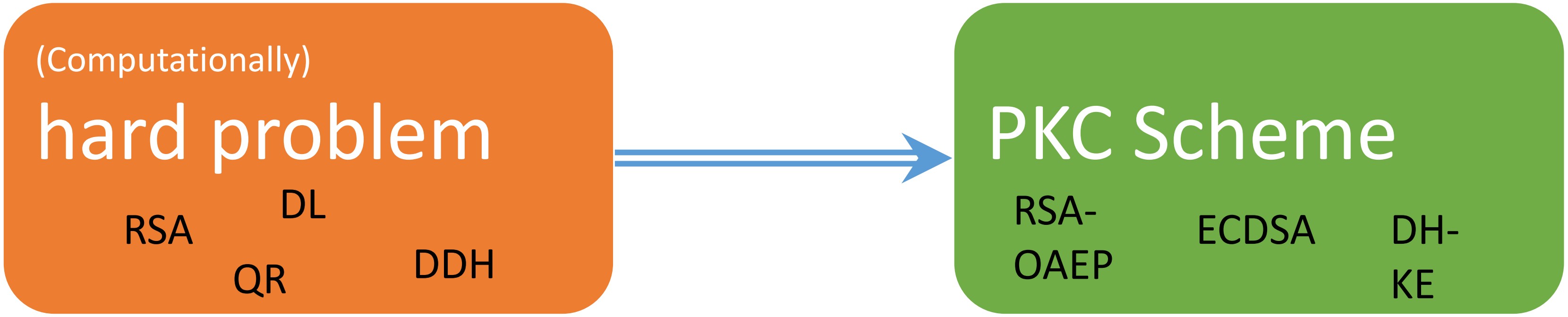
pk, Cert(pk belongs to shop)

PKC to establish shared secret sk

SKC secured communication using sk

The
TEA SHOP
FINE TEAS & TISANES
PURE & BLENDED

How to build PKC



The problem

- Large (few thousand logical qubits) quantum computers can solve previously used problems (Factoring & DLog)
- All previous public key schemes are broken
- No KEX, KEM, PKE, and DSIG

- Symmetric key primitives generally remain secure!

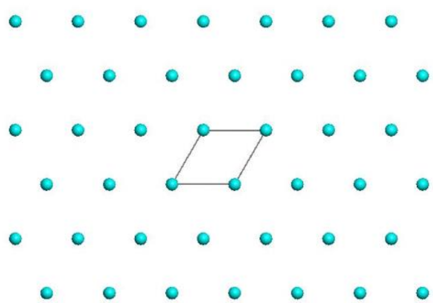
This is a problem that QKD cannot solve!

But post-quantum cryptography can!

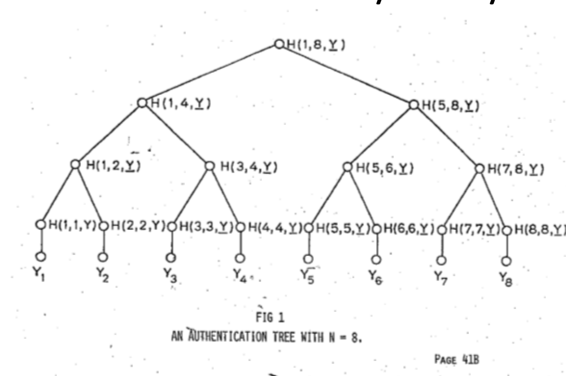
Early post-quantum crypto

„Cryptography based on problems that are conjectured to be hard even for quantum computers.“

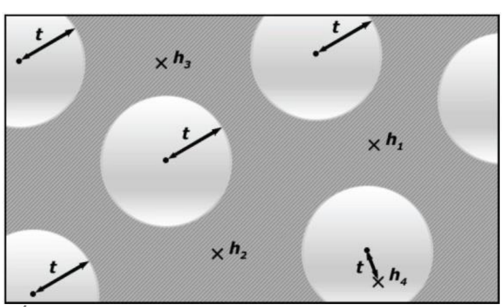
Lattice-based: SVP / CVP



Hash-based: CR / SPR / ...



Code-based: SD



Multivariate: MQ

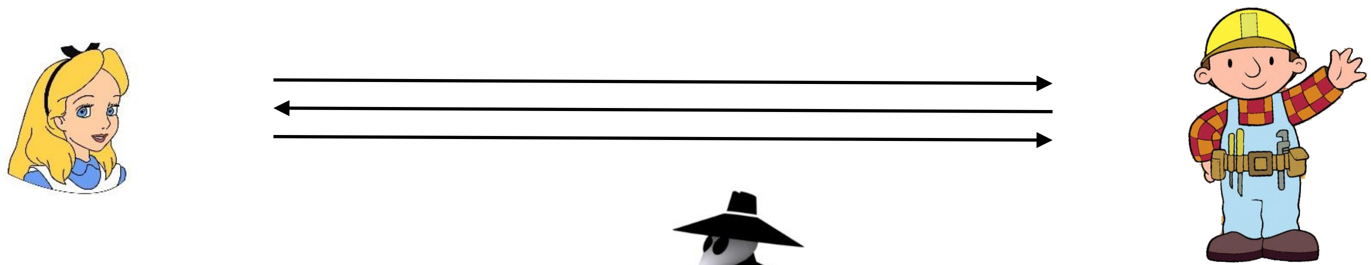
$$y_1 = x_1^2 + x_1x_2 + x_1x_4 + x_3$$

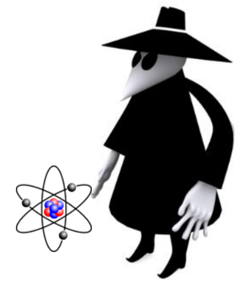
$$y_2 = x_3^2 + x_2x_3 + x_2x_4 + x_1 + 1$$

$$y_3 = \ldots$$

Modern post-quantum crypto

„Users using cryptography on conventional computers facing quantum adversaries“





Adds questions like

- How to argue security?
- Are our security models sound?
- What is the complexity of actual quantum attacks?

The computational complexity approach

- Public key cryptography cannot be information theoretically secure
- We need to base it on hardness of computational problems
- Cryptanalysis needed to determine complexity of solving problems aka breaking systems
  - Needed to select parameters.

Conjectured quantum-hard problems

- Solving multivariate quadratic equations (MQ-problem)
  -> Multivariate Crypto
- Syndrom decoding problem (SD)
  -> Code-based crypto
- Short(est) and close(st) vector problem (SVP, CVP)
  -> Lattice-based crypto
- Breaking security of symmetric primitives (SHAx-, AES-, Keccak-,... problem)
  -> Hash-based signatures / symmetric crypto
- (Finding isogenies between supersingular elliptic cruves
  -> SIDH)

NIST Competition

“We see our role as managing a process of achieving community consensus in a transparent and timely manner” NIST’s Dustin Moody 2018

Status of the competition

- Nov 2017 Submissions collected
- Dec 2017 Complete & Proper proposals published
  - -> Starts round 1 (of 2 or 3 rounds)
- 2022 – 2024 Draft standards exist

Submissions
(69 complete & proper)

Type	PKE/KEM	Signature	Signature & PKE/KEM
Lattice	21 (-1 due to merge)	5	
Code-based	18 (-1 withdrawn)	3 (-1 withdrawn)	
Hash-based		3	
Multivariate	2	7	2 (-1 withdrawn)
Braid group		1	
Supersingular Elliptic Curve Isogeny	1		
Satirical submission			1
Other	4 (-2 withdrawn)		

First evaluation results

Submissions

- Submissions generally follow a few previously known theoretic constructions.
- Submissions differ in how the theoretical construction is implemented

Attacks

- 11 attacks on 10 schemes published.
- No “big surprises” (aka efficient solution to one of the underlying hard problems)
- Attacks either break those schemes that are “fundamentally new” or exploit implementation decisions

The computational problems

MQ-Problem

Let $\boldsymbol{x} = (x_1, \ldots, x_n) \in \mathbb{F}_q^n$ and $\mathbf{MQ}(n, m, \mathbb{F}_q)$ denote the family of vectorial functions $\boldsymbol{F}: \mathbb{F}_q^n \longrightarrow \mathbb{F}_q^m$ of degree 2 over $\mathbb{F}_q$:

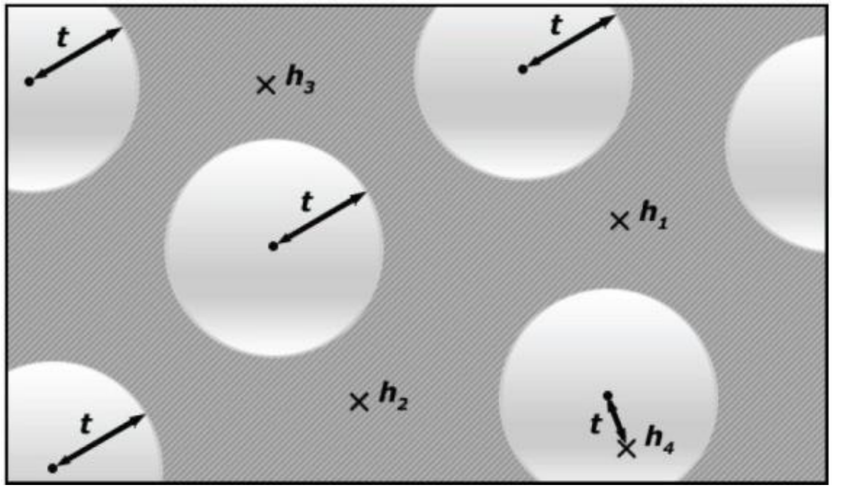
$$\mathbf{MQ}(n, m, \mathbb{F}_q) = \left\{ \boldsymbol{F}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x}) \mid f_s(\boldsymbol{x}) = \sum_{i,j} a_{i,j} x_i x_j + \sum_i b_i x_i \,, \right.$$

Multivariate Cryptography

- First proposal 1988
- Only signatures
  -> (new proposal for encryption exists but very recent)
- Cryptanalysis tasks:
  - Hardness of solving random MQ-instance
  - Hardness of solving “special” MQ-instances
- Known quantum attacks:
  - “Quantization” of classical algorithms (Bernstein & Yang ‘17, Faugère, Horan, Kahrobaei, Kaplan, Kashefi & Perret ‘17)
  - Cost $\mathcal{O}(2^{cn})$, $c = 0.457$ for m=n and q=2

Syndrom Decoding Problem

Given a matrix $G \in \mathbb{F}_q^{k\times n}$ of rank k, the set $C \coloneqq \{mG : m \in \mathbb{F}_q^k\}$ is called a linear code with generator matrix G. If $C = \left\{c \in \mathbb{F}_q^n : Hc^t = 0\right\}$ we call H the parity check matrix.



Syndrom Decoding Problem

Given:

- Linear Code $C \subseteq \mathbb{F}_q^n$,
- Syndrom $s \subseteq \mathbb{F}_q^k$,
- and error bound $b \in \mathbb{N}$

Return:

- $e \in \mathbb{F}_q^n$ of weight $\leq b$ such that $He^t = s$

Decision version is NP-hard (Berlekamp, McEliece & v.Tilborg ‘78; Barg ‘94)

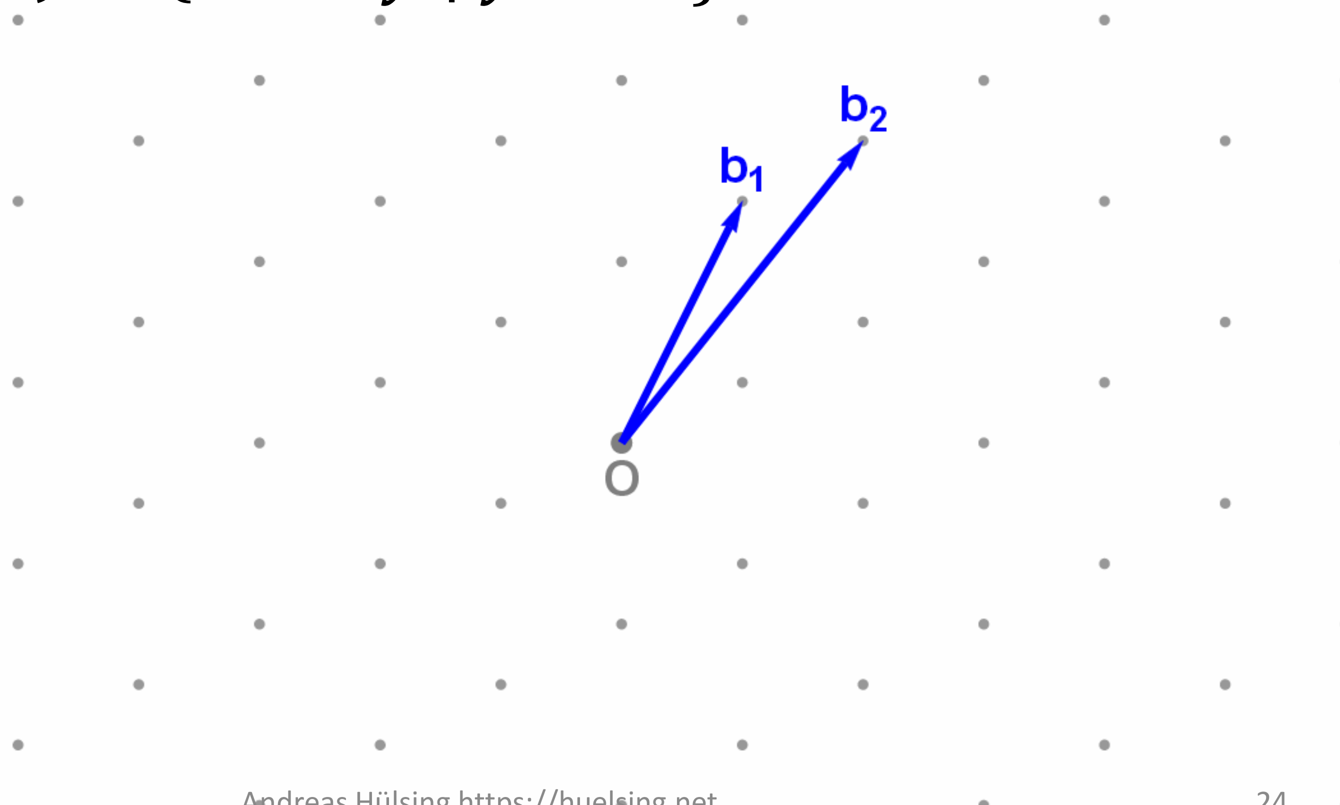
Code-based cryptography

- First proposal 1978: McEliece with binary Goppa codes
- Until recently, practical proposals only known for KEM
- Either huge keys or structured codes (QC-MDPC)
- Cryptanalysis tasks:
  - Hardness of solving random SD-instance
  - Hardness of solving SD for specific codes (QC-MDPC, Goppa)
- Known quantum attacks:
  - "Quantization" of classical algorithms (Kachigar & Tillich '17)
  - Cost $\mathcal{O}(2^{cn})$, $c = 0.058$ worst-case

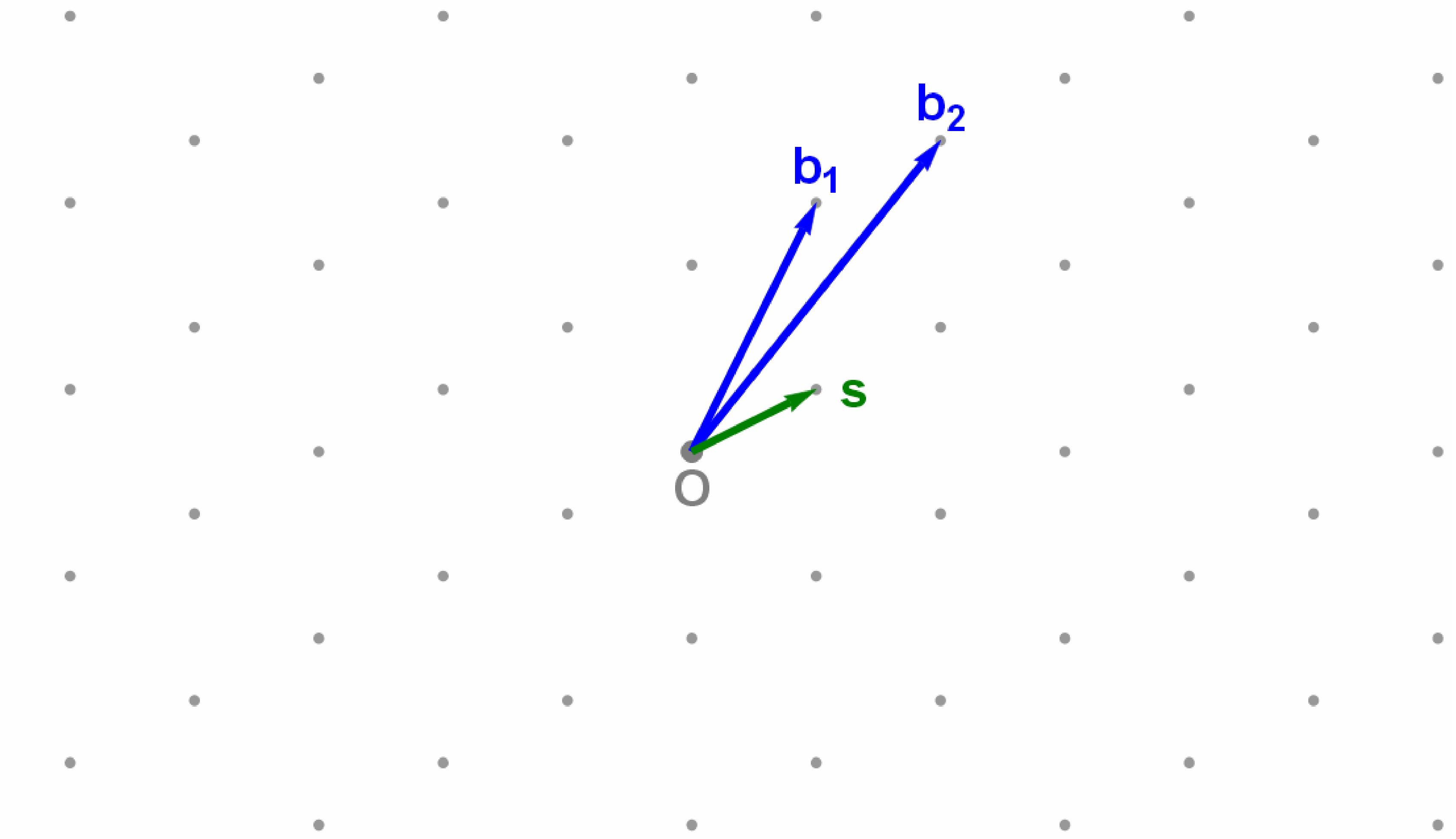
Lattice-based cryptography

Basis: $B = (b_1, b_2) \in \mathbb{Z}^{2\times 2}; b_1, b_2 \in \mathbb{Z}^2$

Lattice: $\Lambda(B) = \{x = By \mid y \in \mathbb{Z}^2\}$



Shortest vector problem (SVP)



(Worst-case) Lattice Problems

- **SVP:** Find shortest vector in lattice, given random basis. NP-hard (Ajtai'96)
- **Approximate SVP (αSVP):** Find short vector (norm < α times norm of shortest vector). Hardness depends on α (for α used in crypto not NP-hard).
- **CVP:** Given random point in underlying vectorspace (e.g. $\mathbb{Z}^n$), find the closest lattice point. (Generalization of SVP, reduction from SVP)
- **Approximate CVP (αCVP):** Find a „close“ lattice point. (Generalization of αSVP)

Lattice-based crypto

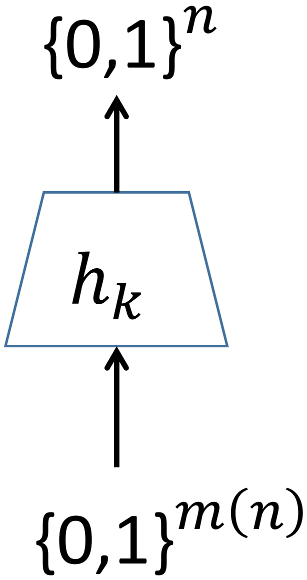
- First proposal GGH (proposed 1995, published 1997) or Ajtai (1996)?
- Signatures & KEM / KEX
- Either huge keys and/or sigs or structured lattices (Ideal / module lattices)
- Cryptanalysis tasks:
  - Hardness of solving αSVP for random lattices
  - Hardness of solving αSVP for structured lattices (Ideal-, Module lattices)
- Known quantum attacks:
  - “Quantization” of classical algorithms (Laarhoven, Mosca & v.d.Pol ‘15; Aono, Nguyen & Shen '18)
  - Cost $2^{cn+o(n)}$, $c = 0.268$ (heuristically)

(Hash) function families

- $H_n \coloneqq \left\{h_k \colon \{0,1\}^{m(n)} \to \{0,1\}^n\right\}$

- $m(n) \geq n$

- „efficient“



Preimage resistance (PRE)

$$H_n \coloneqq \left\{ h_k \colon \{0,1\}^{m(n)} \rightarrow \{0,1\}^n \right\}$$

y_c, k

$\mathcal{A}$

x^*

$$h_k \xleftarrow{\$} H_n$$
$$x \xleftarrow{\$} \{0,1\}^{m(n)}$$
$$y_c \leftarrow h_k(x)$$

Success if $h_k(x^*) = y_c$

Collision resistance (CR)

$H_n := \{h_k : \{0,1\}^{m(n)} \to \{0,1\}^n\}$

$h_k \xleftarrow{\$} H_n$

Success if

$h_k(x_1^*) = h_k(x_2^*)$ and
$x_1^* \neq x_2^*$

k

$\mathcal{A}$ 

(x_1^*, x_2^*)

Second-preimage resistance (SPR)

$H_n \coloneqq \{h_k: \{0,1\}^{m(n)} \rightarrow \{0,1\}^n\}$

$h_k \xleftarrow{\$} H_n$
$x_c \xleftarrow{\$} \{0,1\}^{m(n)}$

Success if

$h_k(x_c) = h_k(x^*)$ and
$x_c \neq x^*$

x_c, k

$\mathcal{A}$

x^*

Hash-based signatures

- First proposal Lamport (1979)
- Only signatures
- Fast & compact (2kB, few ms), but stateful, or
- Stateless, bigger and slower (41kB, several ms).
- Cryptanalysis tasks:
  - Solving PRE, SPR, CR,… for random function families
  - Solving PRE, SPR, CR,… for specific hash function (SHA2, SHA3)
- Quantum attacks:
  - Upper & lower bounds for generic attacks (Zhandry ‘15, Huelsing, Song & Rijneveld ‘16)
    - PRE, SPR: $\Theta(\frac{q^2}{2^n})$, CR: $\Theta(\frac{q^3}{2^n})$
  - Costs in more realistic models are worse (e.g. Bernstein & Souza Banegas ‘17)

Quantum cryptanalysis?

All known algorithms improve conventional algorithms by **less than a square root** factor!

Conclusion

- We need more actual quantum cryptanalysis!
- Skipped due to time: There are a lot of open questions beyond selecting new DSIG / KEM / PKE schemes:
  - What are the right models when proving security?
    - See notion of collapsing [Unruh ‘16], or the ongoing discussion about indifferentiability [Zhandry ‘18, *Carstens, Ebrahimi, Tabia* & Unruh ‘18]
  - How do we proof security in these models?
    - Real-Ideal: We often do not even know quantum complexity in ideal setting

Resources

- PQ Summer School: https://2017.pqcrypto.org/school/index.html
- NIST PQC Standardization Project: https://csrc.nist.gov/Projects/Post-Quantum-Cryptography

PQCrypto

PQCRYPTO ICT-645622

Thank you!
Questions?